DENSITY IN VARIATIONS ON MOMOTANI BRICK WALL

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In [V1] and [V2], I looked at variations on Momotoni's [M] basic brick wall tessellation, by looking at possible changes in directions of the folds. Recall we looked at the crease lines as in Figure 1

In this article, instead of varying the directions of the crease lines, I will change the dimensions of the rectangles in the crease pattern. I will investigate how the "density" of the origami changes as the rectangle varies.

Suppose the rectangle has side lengths $a > b \geq 0$. If we keep the same symmetries of the pattern, then the rest of the pattern is determined by a, b. Properties of the origami that we're intersted in remain unchanged as long as $r = a/b$ remains unchanged, so we label the origami by r.

Some examples are shown in Figure 3 (I made these in Findhorn and on the train on the way up there - 10 hour journey!). Note that these have the same appearance both sides.

Note that the case $r = 2$ is the usual brick wall of Momotani [M]. For r with $2 < r \leq 3$, we obtain a flat folding origami, but the bricks overlap like tiles.

Note that the case $r = 3$ is the limit of being able to fold flat; when $r > 3$, the origami can't fold flat with these crease directions, unless parts on opposite sides of the long side of brick pass through each other. However, if different directions of the creases are chosen – as for example the "cascade" pattern of [V1], then it does fold flat. See Figure **??** and Figure 6 for an example, where $r = a/b = 5/1 = 5$. I really like the texture of this pattern! Hard to capture in the photos!

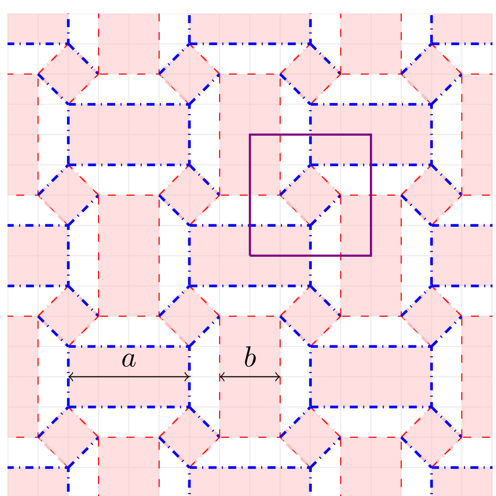


FIGURE 1. crease lines for brick wall [M], with $a = 4, b = 2$

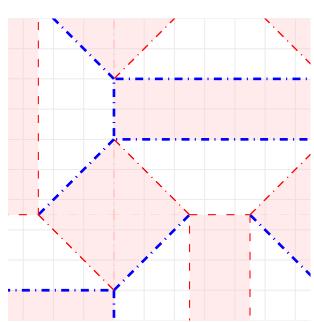


FIGURE 2. brick pattern

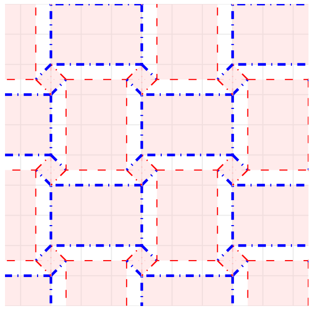
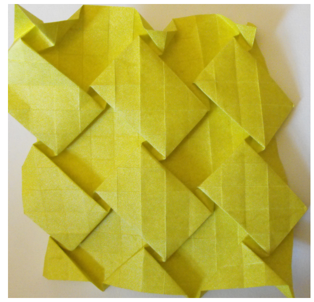
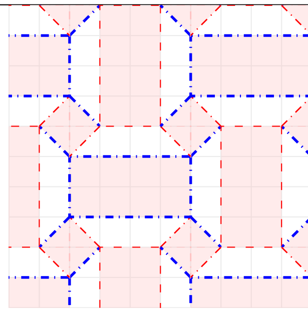
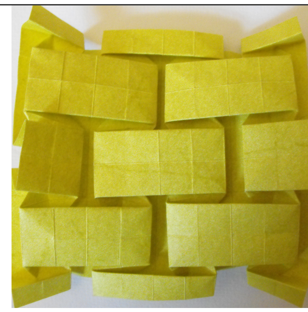
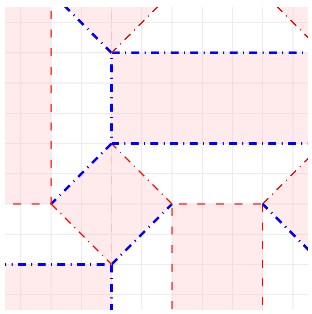
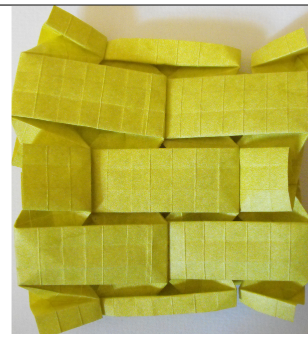
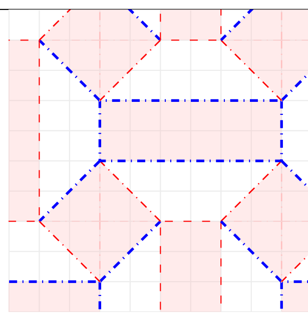
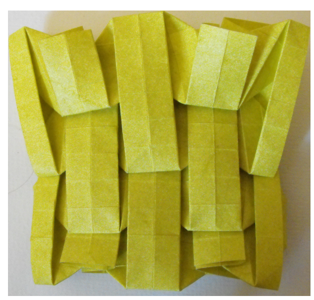
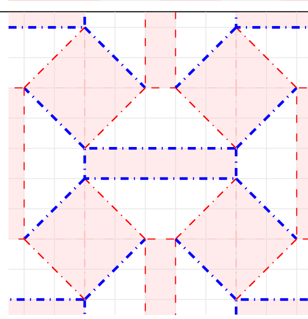
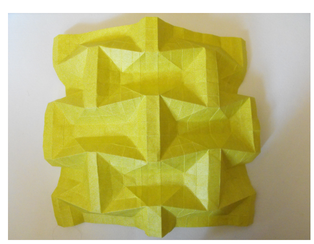
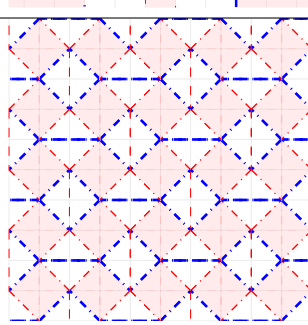
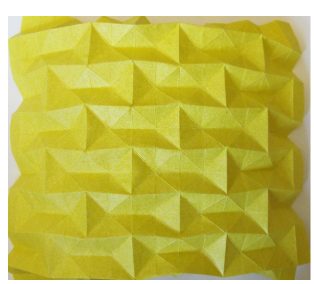
$r = a/b$	crease pattern	folded
1.5=3/2		
2=4/2		
2.33=7/3		
3=6/2		
5=5/1		
$\infty = 2/0$		

FIGURE 3. Wall variations

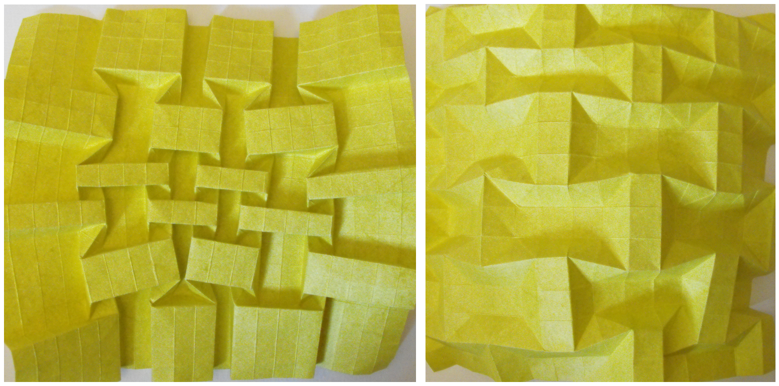


FIGURE 4. Brick patterns with varying brick size

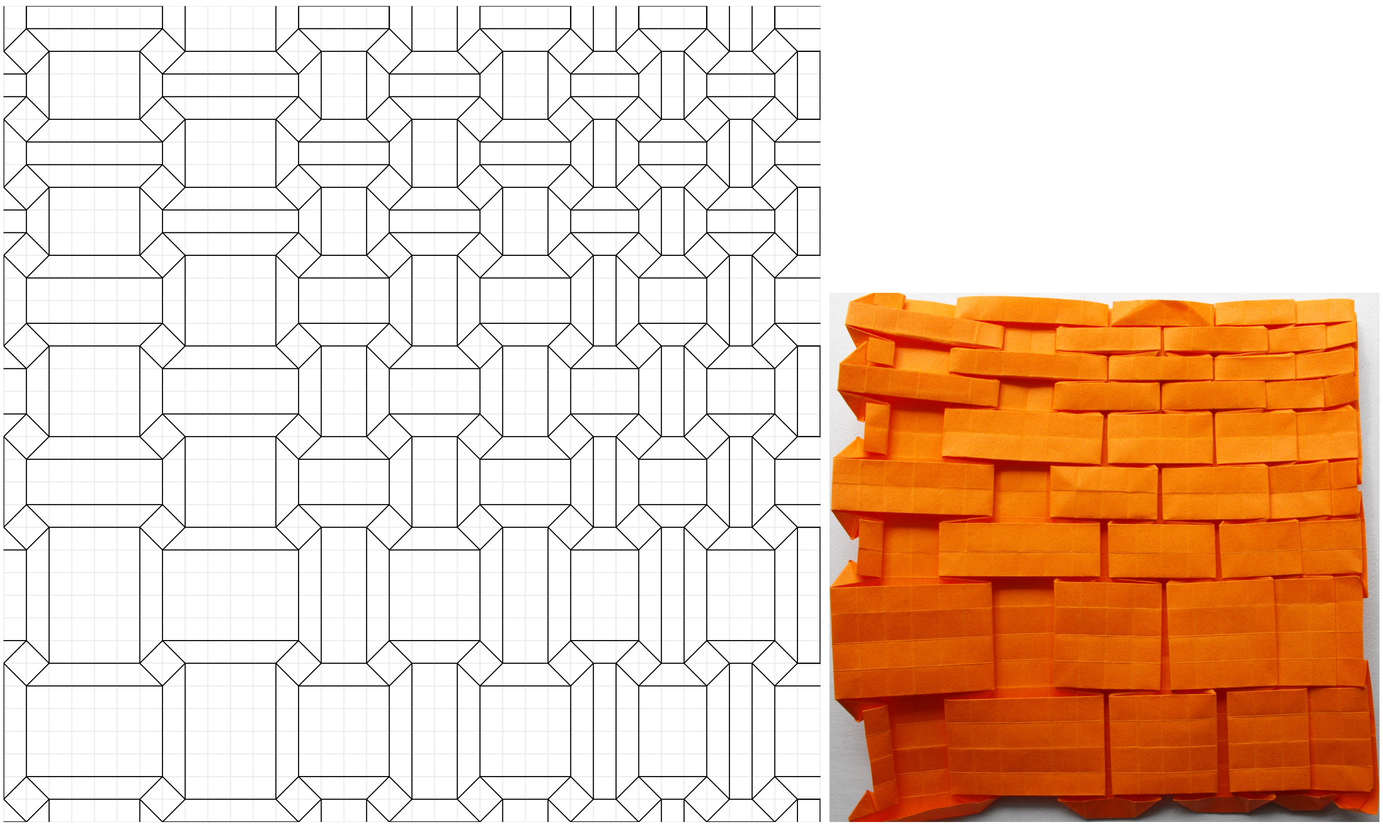


FIGURE 5. Varying brick size/density example, crease pattern and origami

However, even when $r > 3$, we can still make the creases, just not fold flat, as in the last two rows in the table in Figure 3; we also need to make creases across the diagonals of the squares in the pattern.

Note that for all these crease patterns, we could change the crease directions as in the patterns in [V1] to obtain variations on these origamis.

1. Vary size bricks

We can also fold patterns where the brick sizes vary, for example, as in Figure 4. Figure 5 shows a crease pattern for a brick wall origami which transitions from having $r = 1.5$ in the lower left, to $r = 3$ in the upper right.

2. Density

Note that in Figure 3, as r increases, the "density" of the tessellation increases. By this, I mean the amount of paper folded on top of itself in the folded origami. To be precise, let

$$d_1(r) = \lim_{\text{paper size gets big}} \left(\frac{\text{area after folding brick wall pattern with parameter } r}{\text{area of sheet of paper before folding}} \right)$$

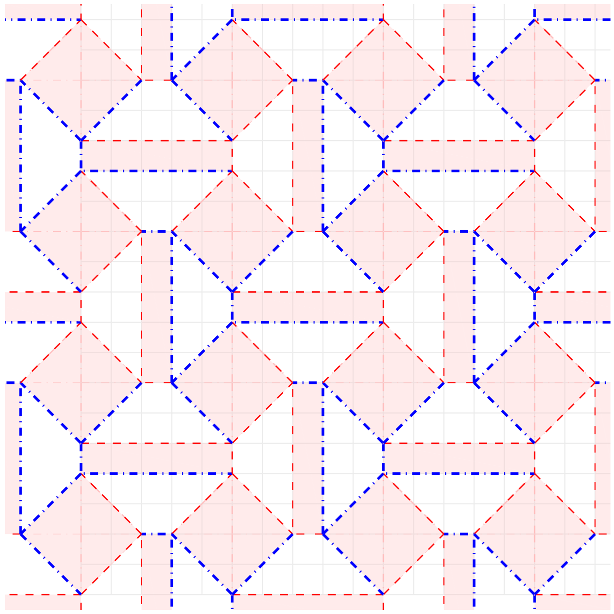


FIGURE 6. Foldable crease pattern when $r = 5$

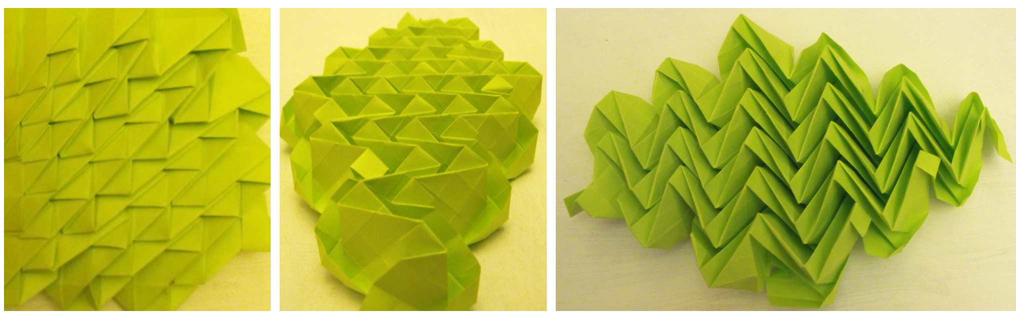


FIGURE 7. Folded to alternate crease pattern when $r = 5$; several photos, to show flat (first photos) and expanded

Another density can be defined as the amount of the crease pattern coloured pink in the diagrams where

$$d_2(r) = \lim_{\text{paper size gets big}} \left(\frac{\text{pink area of paper in pattern with parameter } r}{\text{area of sheet of paper}} \right)$$

In Figure 1, a unit square of the tessellation is outlined. The width/height of this square before folding is $a/2 + b/2 + (a-b)/2 = a$ The width of the white trapezium is $(a-b)/2$, so after folding the width is $a - (a-b) = b$ So,

$$d_1(a,b) = \frac{b^2}{a^2} = 1/r^2$$

The area of the white part of this square is $(a+b)(a-b)/2$ So

$$d_2(a,b) = a^2 - (a^2 - b^2)/2 = (1 + 1/r^2)/2$$

In particular:

$$d_2 = (1 + d_1)/2$$

Probably, d_1 is not the right quantity to measure, since it's not really the right idea of density. Maybe $D_1 = 1/d_1$ would be better. I will investigate in another article whether this relationship holds for other origami tessellations, or what kinds of relationships do hold.

REFERENCES

[M] Momotani 1984 British Origami Society Convention Book
[V1] variations on square twist/Momotani brick wall
[V2] More variations on square twist/Momotani brick wall