Bridges 2024 talk notes

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1.
   - Slide 1 has an image created by the program at
     http://www.mathamaze.co.uk/Truchet3/hexgrid.html
     The programs used to create the images can be found at
     http://www.mathamaze.co.uk/Truchet3/
     - In theory, you should be able to recreate these images and vary parameter,
     - BUT: Note: the programs might not work properly on all devices, so images might not look exactly the same on all devices, and in worst cases may just look blank. I used a mac book, and Chrome or Safari to run the programs for these images. The code is all javascript / webgl.
     - If the program does not display well on your device you can click the controls button so the controls are not over the image.
     - There are lots of preset pictures, which you can bring up with the radio buttons, then alter with other controll buttons
     - some dots can be moved for some variation of parameters; these may only work with mouse, not touch screen.
   - This image combines a curve closely related to the Terdragon fractal curve, a paper folding curve, related to Heighway's fractal dragon curve
   - this image also contains a tesselation of Koch snowflake curves
   - This is the image at the beginning of my Bridges paper
   - I am going to talk about how this image is created, etc
   - All these slides are at www.mathamaze.co.uk/Truchet3

2. Slide 2 is just the image to see more clearly; this is a terdragon curve variation over a Koch snow flake variation

3. Slide 3 gives a summary of the talk; last year I covered fractals from Hinged Truchet Smith tiles; this year I look at hexagons and triangle tilings.

   The idea is to

   (a) Take a hingable tiling
   (b) put a Truchet design on each tile (these are pictured at the top of the page)
   (c) hinge the tiling to an open position where it looks like a version of the initial tiling
   (d) insert new Truchet tiles
   (e) Repeat

(f) Observe the appearance of a fractal curve.

The curves we get for the square, hexagon, triangle 1 and triangle 2 tilings are:

i. Heighway dragon (and relatives)
ii. Terdragon dragon (and relatives)
iii. Sierpinski triangle (as a curve) (and relatives)
iv. Terdragon dragon boundary (and relatives)

Note that the "relatives" correspond to different infinite binary strings.

4. Slide 4 is to recall the square case; I may also demonstrate using a program at

   www.mathamaze.co.uk/GA2021/index5.html

   This material was covered in last year's talk and supplementary material:

   https://archive.bridgesmathart.org/2023

   Important points: although this is motivated by hinged tilings that can be physically realised, we scale down by a factor of $\sqrt{2}$.

   Also, we stop at 45° rotation of tiles, and the background becomes foreground tiles, so we can start the hinging over again.

   With the truchet design, we end up creating Heighway dragon curves

5. Slide 5 shows examples from last year. We don't need to look at these for long

6. Slide 6: Now we consider how to repeat for other hinged tilings, and give some examples to try with.

7. Slide 7 shows all three hinged tilings on one slide. This is just plane tiles, single colour for each tile.

8. Slide 8 Recalls the tile designs; Truchet / Smith design.

9. Slide 9 is about the continuous operation vs the discrete operation. This is to help explain how we can talk about e.g., application of 3.5 operations.

10. Slide 10: shows the continuous and discrete steps of the operation, with some info about colouring rules, and examples of applying up to 3 operations.

    Steps for discrete replacement rule:

    - start with the hexagon tiles
    - apply the replacement rule for each tile. I put images over each other to go back and forth to see the change.
    - Remove the original tile outlines, so a new hexagon tiling can be seen.
    - Repeat the process
    - The slides show these steps from 0 to 1, 1 to 2, 2 to 3, then all four, i.e., t=0, 1, 2, 3 together. the number t means how many times the replacement operation has been applied to the whole image.

11. Slide 11: Triangle 1 case

12. Slide 12: Triangle 2 case (less detail on the triangle cases; hopefully the extra infor for the hexagon case helps to make these more comprehensible by comparison)

13. Slide 13 shows the Truchet design on each of the three cases in turn. It's shown in each case for

(a) individual tile, one replacement operation
(b) several tiles, one replacement applied to all
(c) result of many replacements, resulting in fractals

The resulting fractals are:

(a) hexagons ⇒ Terdragon variants; these are fractiles; we can look at the program to see tilings
(b) Triangles 1 ⇒ Sierpinski; tiling of many differnt sizes of this
(c) Triangles 2 ⇒ Terdragon variants boundary

We can also compare the hexagon and triangle 2 case, to see their relationship

14. Slide 14 shows how to use a different colouring procedure to obtain the Koch snowflake. Slide 14 shows the rule

15. Slide 15 shows application of the colouring rule explained on slide 14

16. Slide 16 shows an example of the Koch snowflake colouring of the background, and a Terdragon in the foreground.

17. Slide 17: Gives a list of further topics which are contained in the Bridges paper, or the supplementary paper, or a more recent related arXiv paper inspired by the contents of this talk.

    - Varying iteration level across the image: easy to do with webgl, can give some nice effects
    - In order to prove the curves are the given fractals, we use L-systems, which describe the curves by a string of symbols.
    - The reason the triangle 2 gives boundary of the terrdragon case can be understood by considering how these tiles are related
    - Extending the relationship to the square case gives rise to L-systems for the boundary of the Heighway dragon and relatives, and ends up solving some problems of 50 years standing. This is written up in some arXiv papers and other javascript programs