Modular origami unit for polyhedra, vertex degree 4

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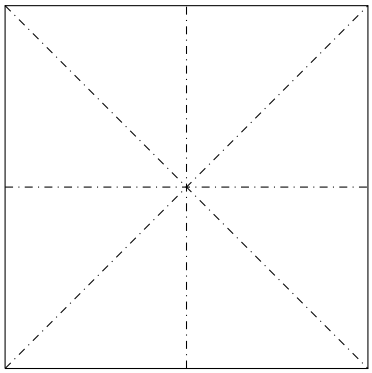
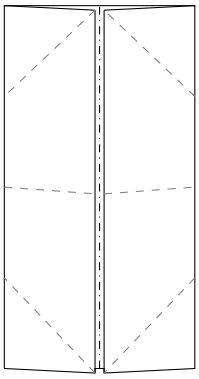
Instructions –

A. Tedious bit - precreasing to get a neat result. Skip to step 10 to see the result of this, and find another method to get there if you prefer!

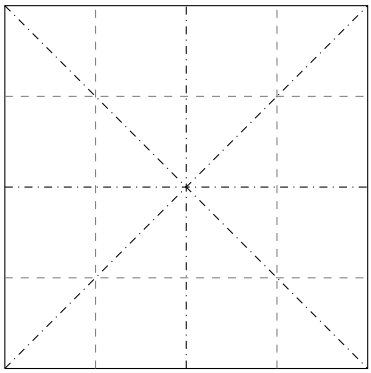
1) Start with a square of paper. fold in half in four ways, opening out after each fold, to obtain the following creases:



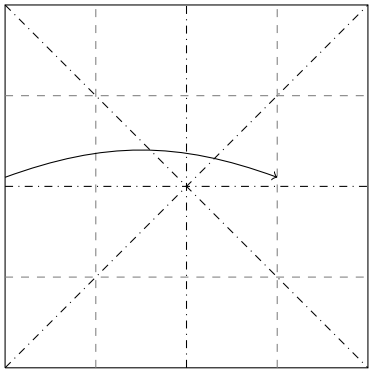
2) Fold opposite edges to the middle (these creases are just for reference):



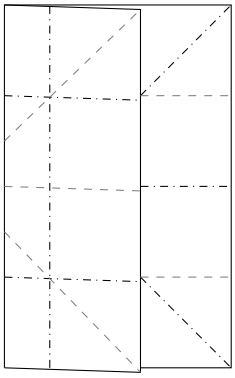
3) open out, fold the other pair of opposite edges together, open out, to obtain:



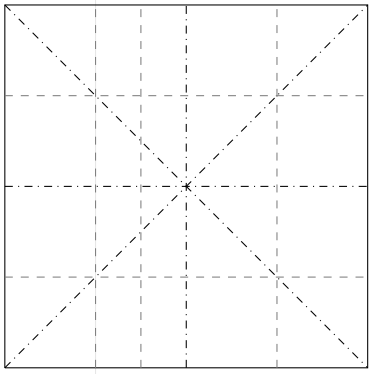
4) Fold an edge to one of the creases just made:



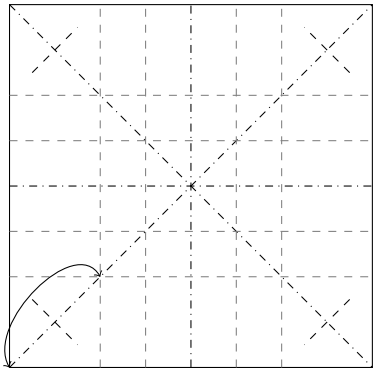
Fold...



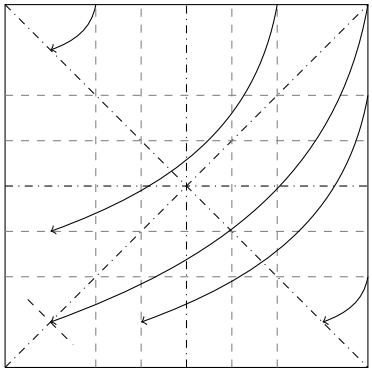
...then open out. Observe the new crease



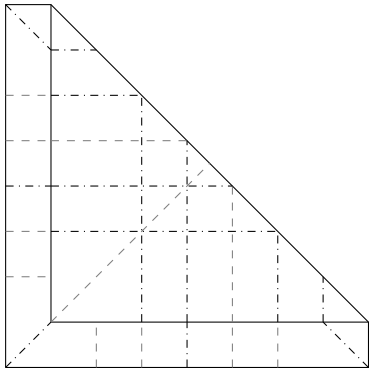
5) Repeat step 4) with the other 3 edges, to obtain these creases. It may also be helpful to add the pinch points as shown



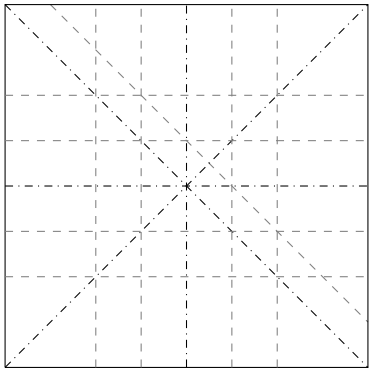
6) Fold a diagonal crease, by matching up these creases, and corner to pinch point



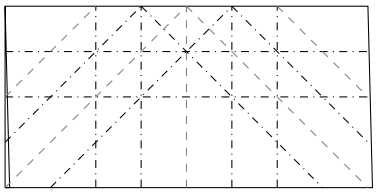
folded...



...unfolded

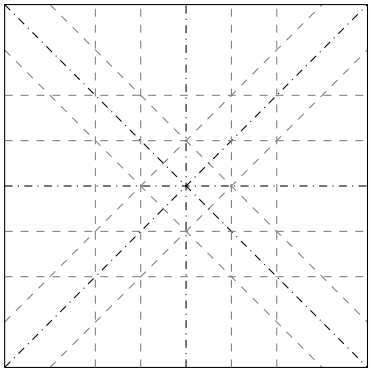


...resulting creases:

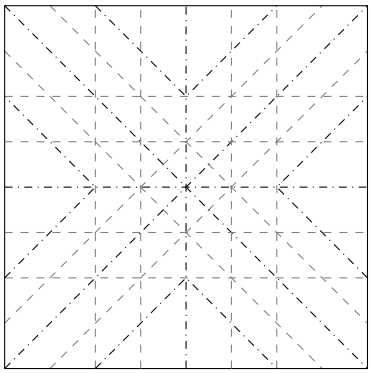


That's the end of the precreasing

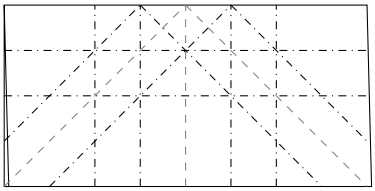
7) Fold 3 more creases similarly



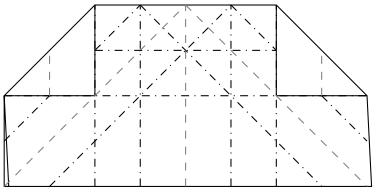
10) Open out, fold in half in other direction and repeat step 9). Open out to this stage:



8) Fold in half, edge to edge:



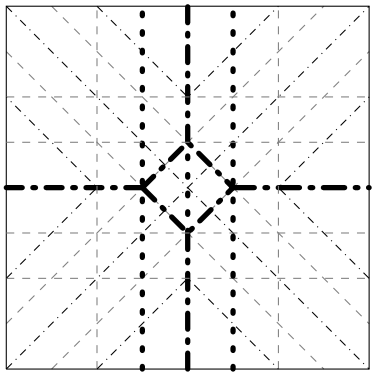
9) then fold as follows:

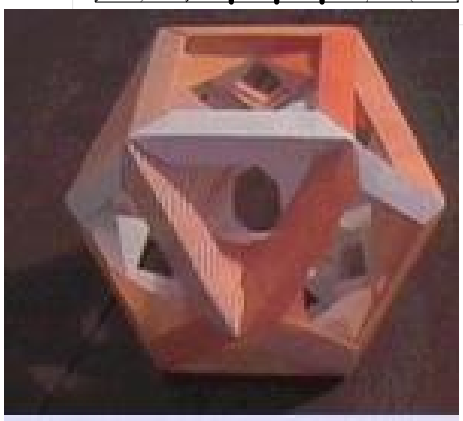


This octahedron uses 6 units, and has some extra tetrahedra inserted

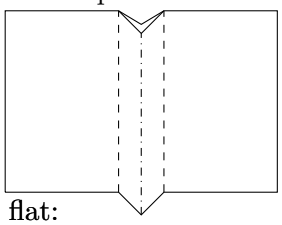
B. Square twist:

11) Make the thick lines clearly valley or mountain folds, plain side up:

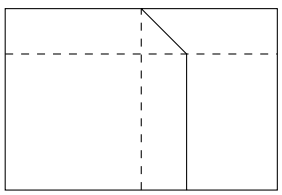




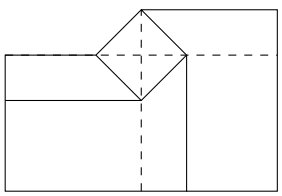
fold in half and push together the middle part:



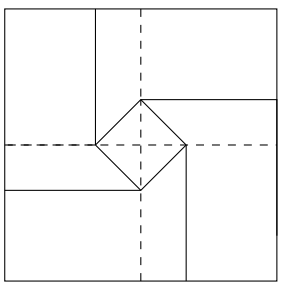
fold flat:



fold down left upper flap:

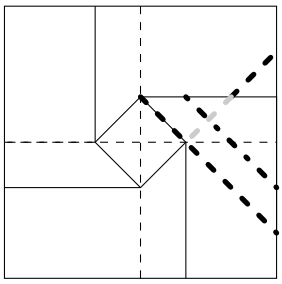


12) fold up the back layer:

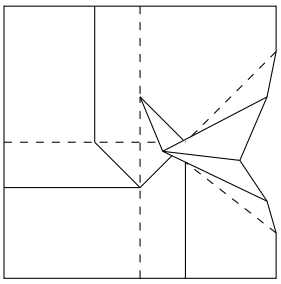


C. Finishing the unit:

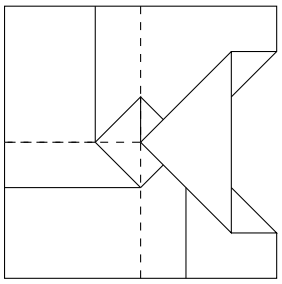
13) We're going to lift up a flap, fold along the marked creases....



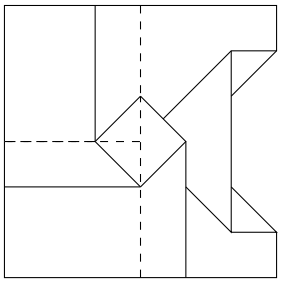
lifting...



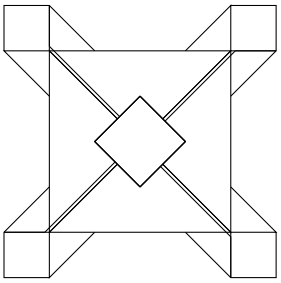
... and squash flat:



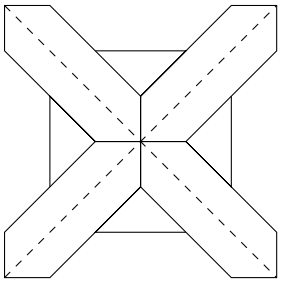
14) tuck under the middle square:



15) Repeat steps 13 and 14 for the other 3 sides:

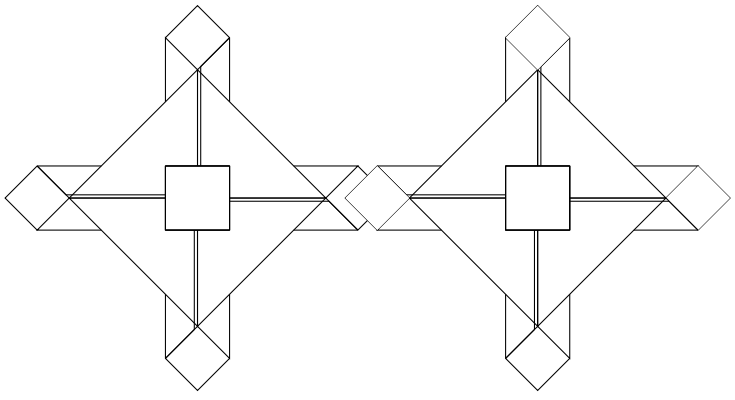


16) Turn over, and make mountain creases on diagonals. The unit is finished!

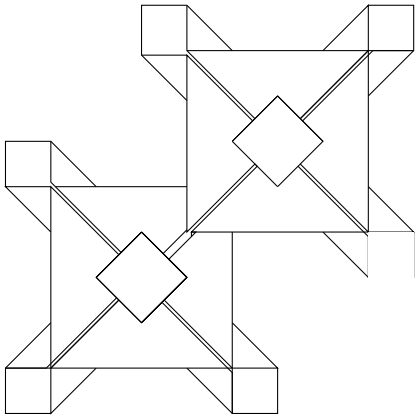


D. Slotting together:

put one "leg" of one unit into the leg of another, and push together



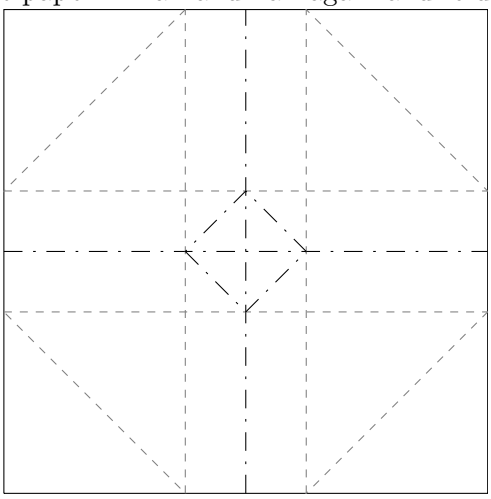
On the reverse, the point of one leg should touch the middle of the other unit.



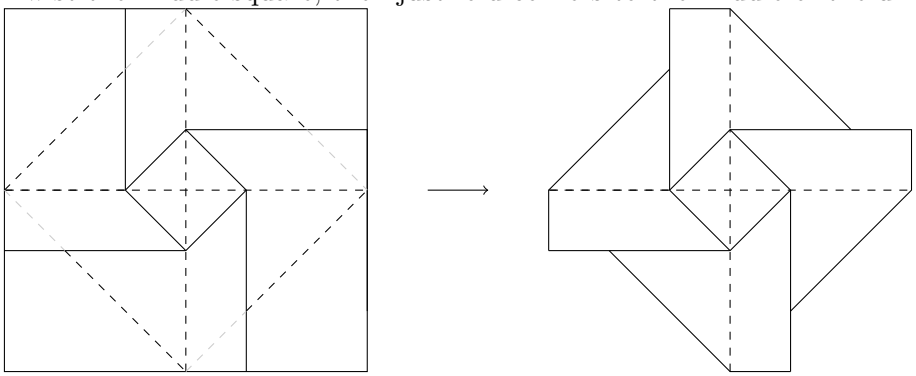
To make an octahedron, you will need 6 of these units. They have to be slotted together to make triangles of legs of three adjacent units. When slotting them together, it can be helpful to temporarily flatten out the legs. There is no other locking, the polyhedron will hold in shape once it's all together. For each unit, I generally put two opposite legs as the ones that go inside other legs, and the other two to be the outside ones.

Very simplified version

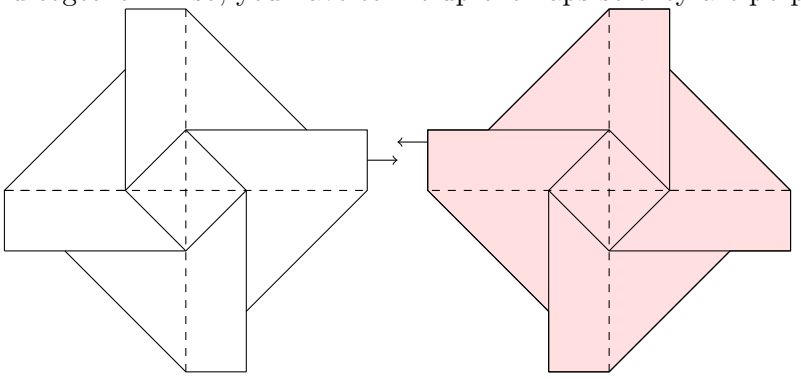
Make the following creases: (fold the paper in quarters both ways first; then fold trhe 3/8 and 5/8 lines; fold the paper in half and half again and fold the middle corner over to get the crease lines of the middle square.)

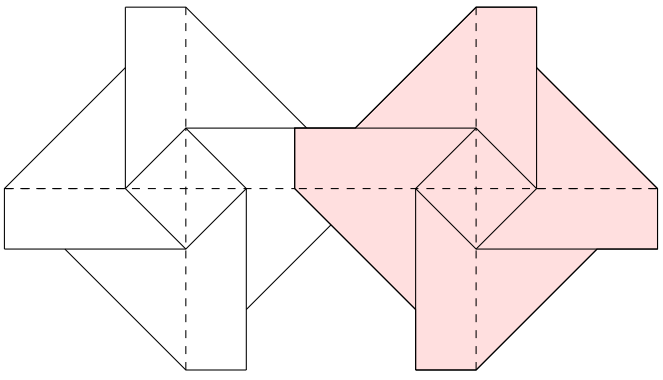


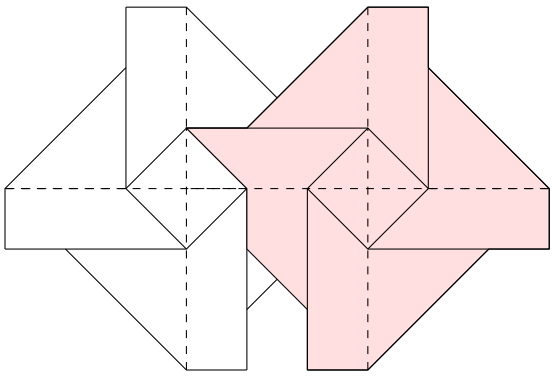
Twist the middle square, then just fold corners to the middle of the unit, tucking them under the flaps.



Two units can slide together, but you will need to **cheat and staple** (horrors!) them together for the unit to hold together. Also, you have to lift up the flaps so they are perpendicular to the rest of the paper.



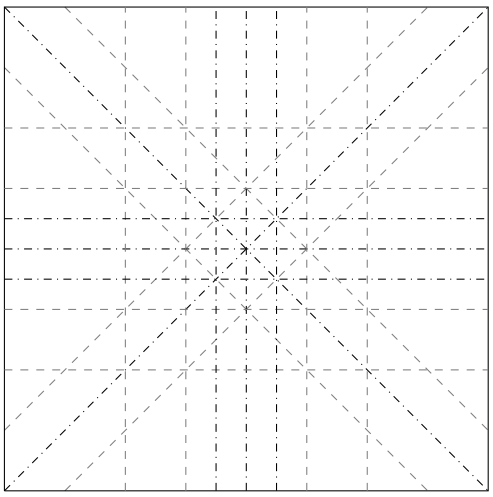




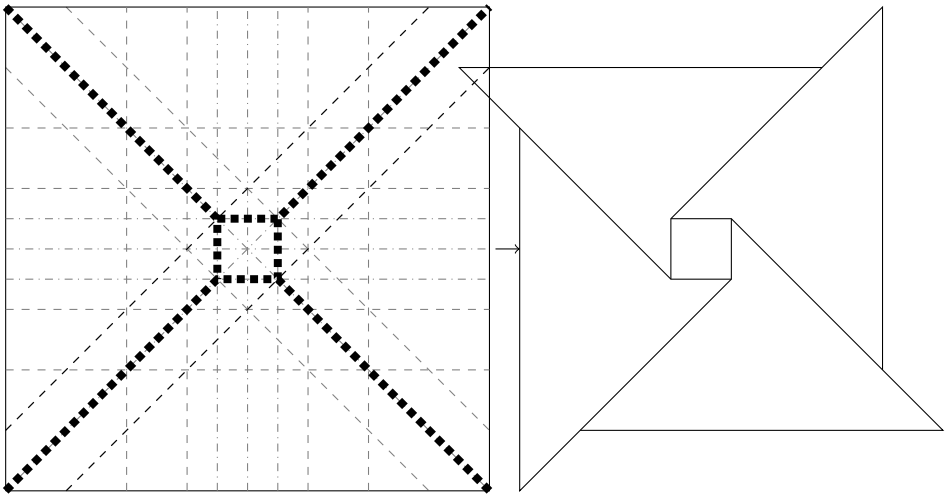
Alternate version, filled in faces

For this version, the paper is rotated through 45°, and then the folding is essentially the same.

A: Creases to fold to begin with:

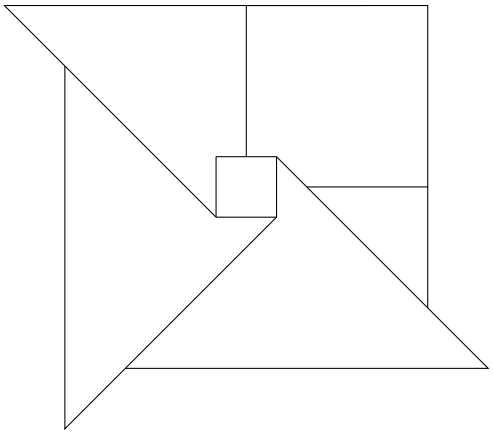


B: Twist the middle square:

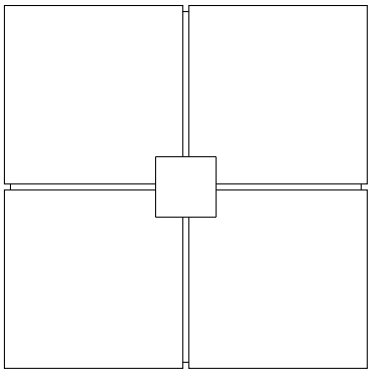


C: Squash flaps, and tuck under central square:

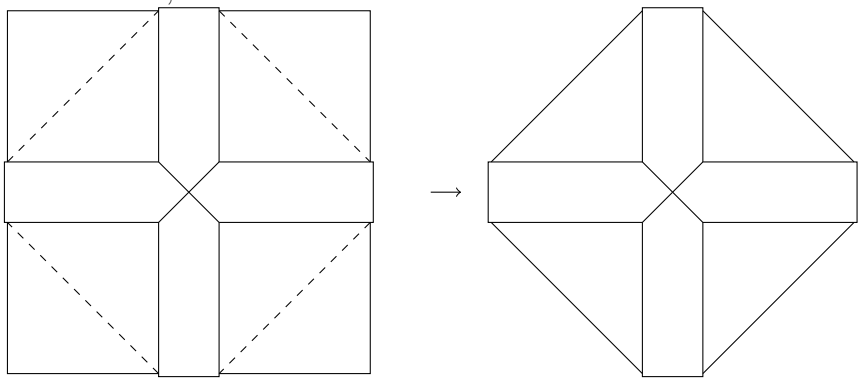
one squashed flap



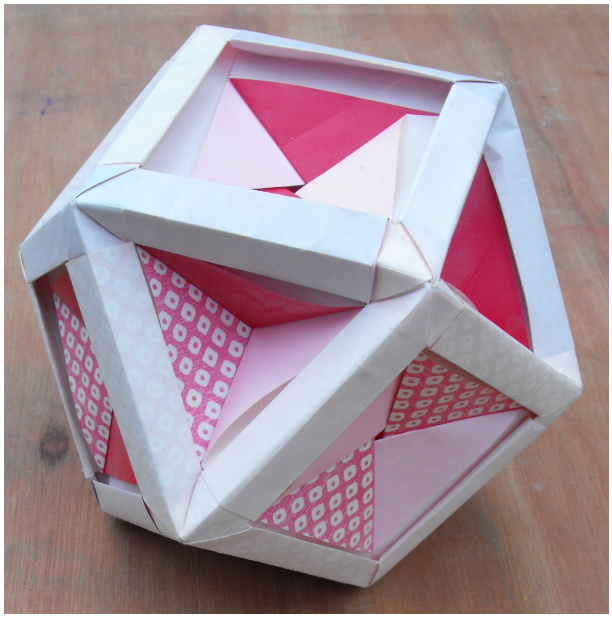
all four flaps opened up and squashed down



D: turn over, and corners to middle:



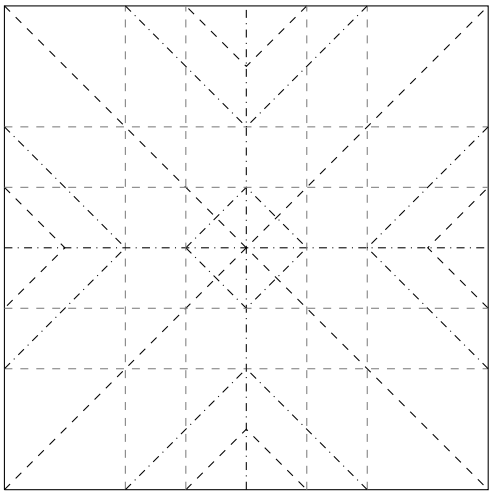
E: Slot together, in same way as other models.



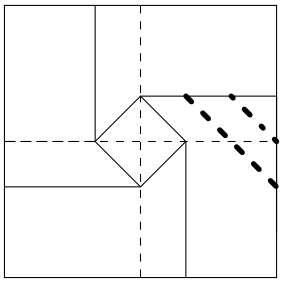
Another alternate version, wider edges, easier to slot togeether

These units fall apart more easily, so use paper clips to hold in place while putting together.

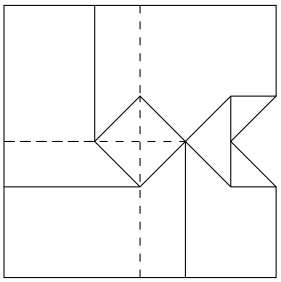
A: Make the following creases:



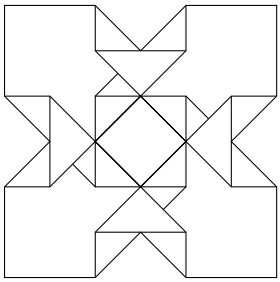
B: Twist middle square as before, and lift up a flap, fold along the marked creases....



... lift and squash flat:



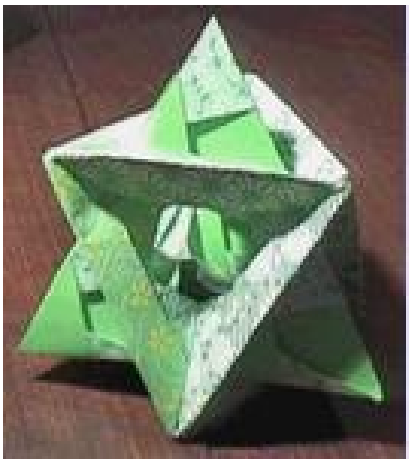
Repeat for other flaps:



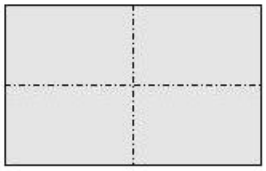
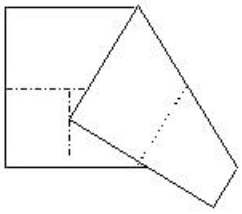
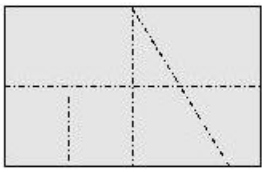
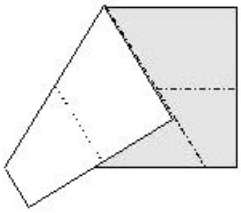
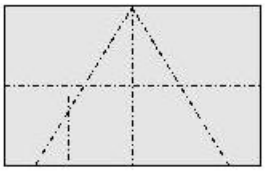
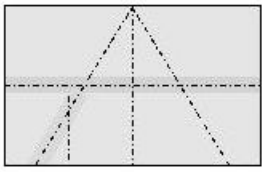
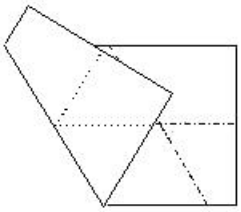
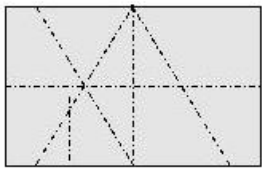
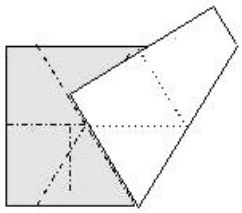
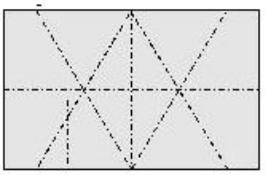
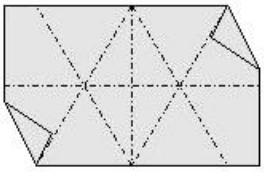
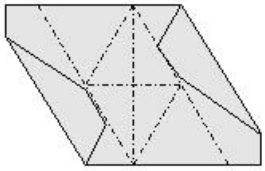
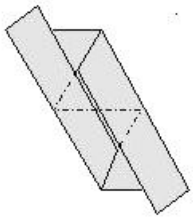
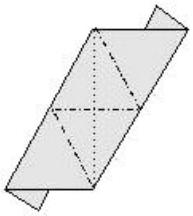
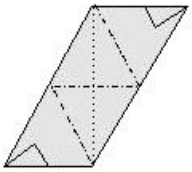
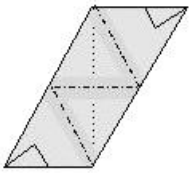
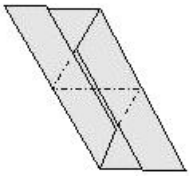
C: Turn over, crease diagonals, and slot together as for other units.

Accessories: units to fit in corners

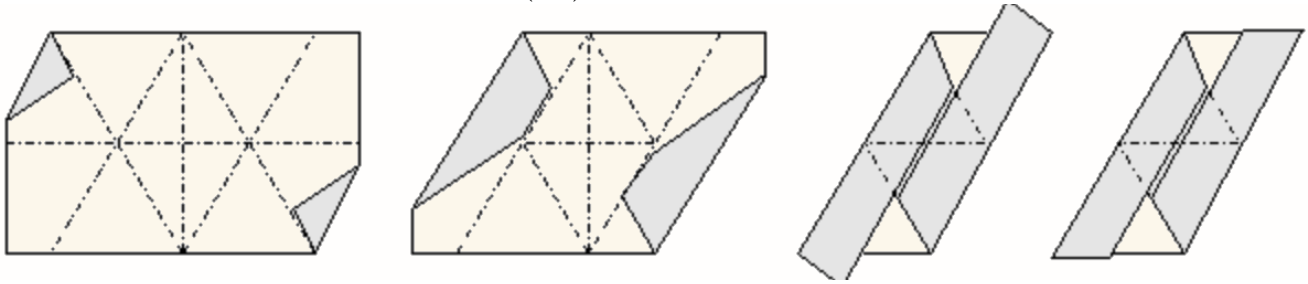
Polyhedra made of Sonobe units, and a tetrahera unit, can be slotted into the corners, transforming our polyhedra to look like other polyhedra, and investigate relationships between them.



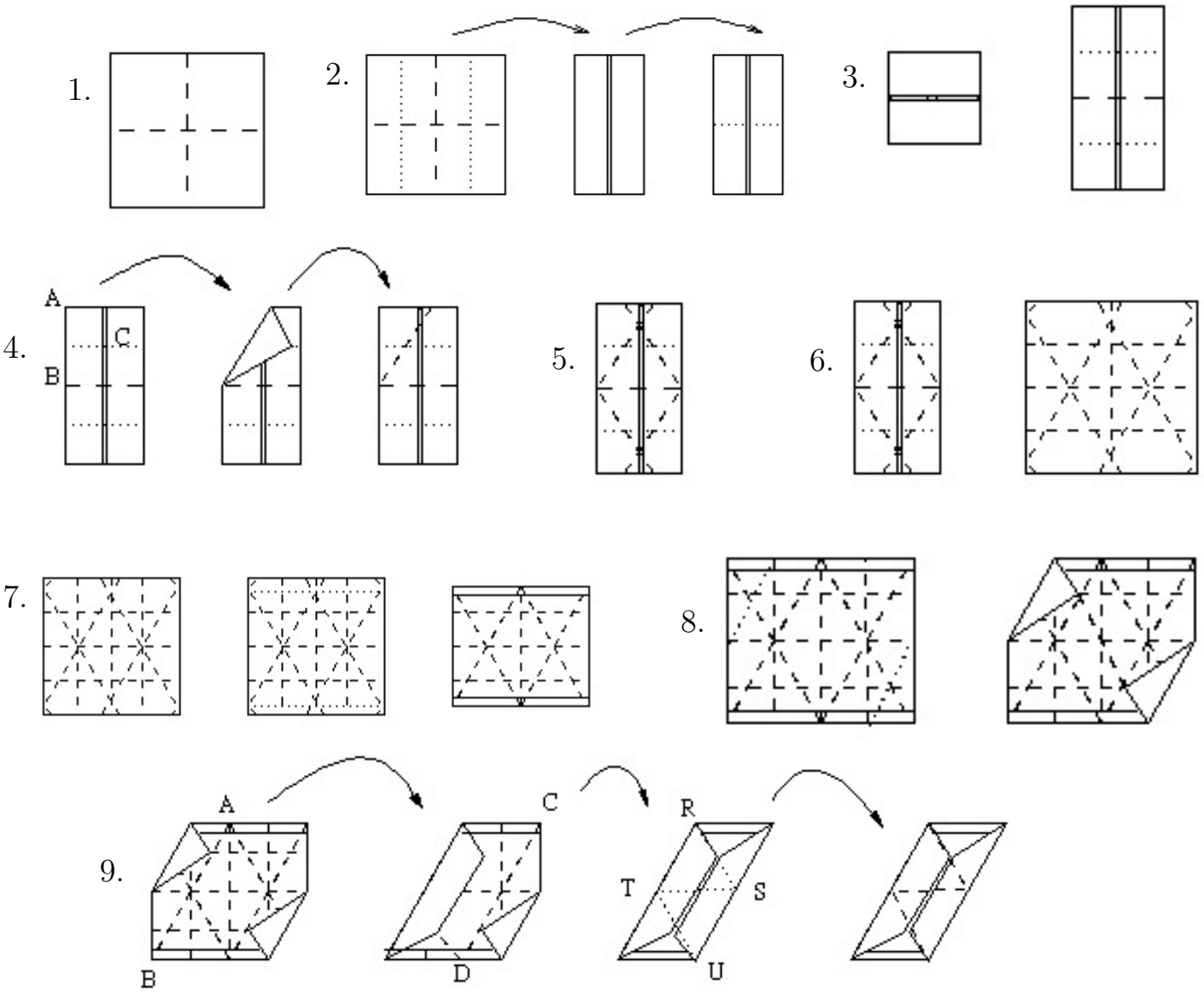
0.1. **Tetrahedron unit.** square for polyhedra : height of rectangle $= 1 : \frac{1}{4}\sqrt{6} \sim 0.6123$
From a rectangle:

1. 
2. 
3. 
4. 
5. 
6. 
7. 
8. 
9. 
10. 
11. 
12. 
13. 
14. 
15. 
16. 
17. 
18. 
19. 

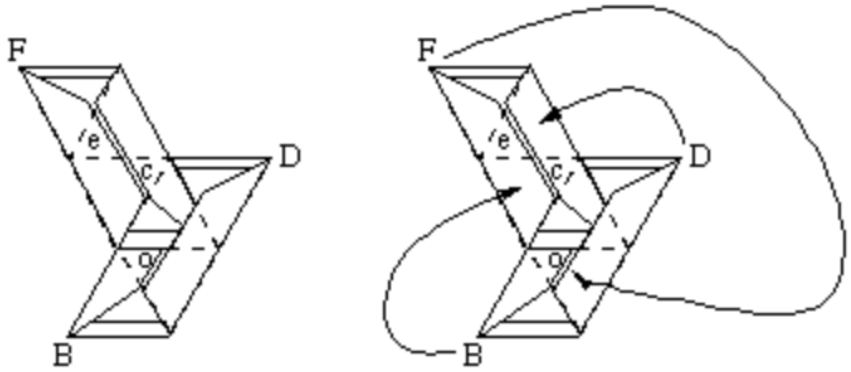
Fold the mirror image, from step (12):



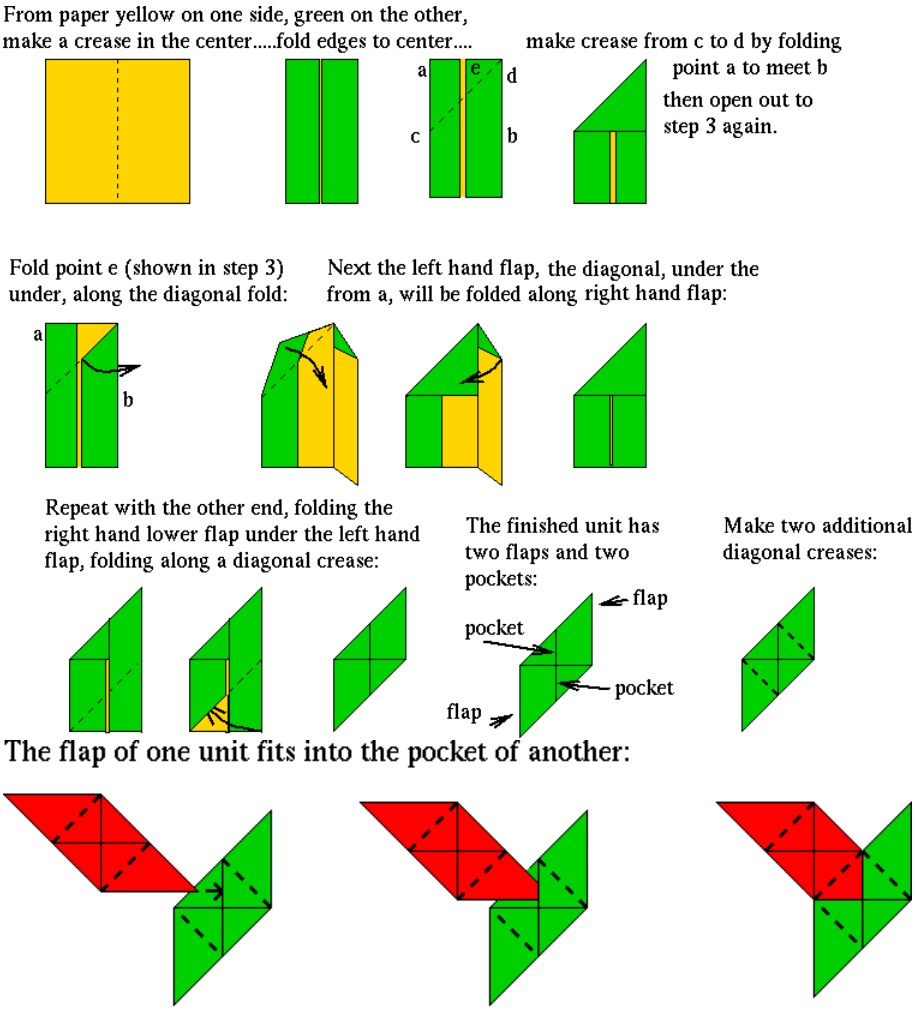
From a square: (square for polyhedra: square for this $= 1 : \frac{1}{2}\sqrt{2} \sim 0.707$)



To make a tetrahedron, take two units, mirror images of each other:



0.2. **Sonobe unit.** Three make a hexahedra: Paper size: square for polyhedra unit: square for sonobe unit $= 1 : \frac{1}{2}\sqrt{2} \sim 0.707$



Notes on degree 4 vertex polyhedra

If V is number of vertices, E number of edges, and F number of faces of a polyhedron, Eulers formula says:

$$V - E + F = 2$$

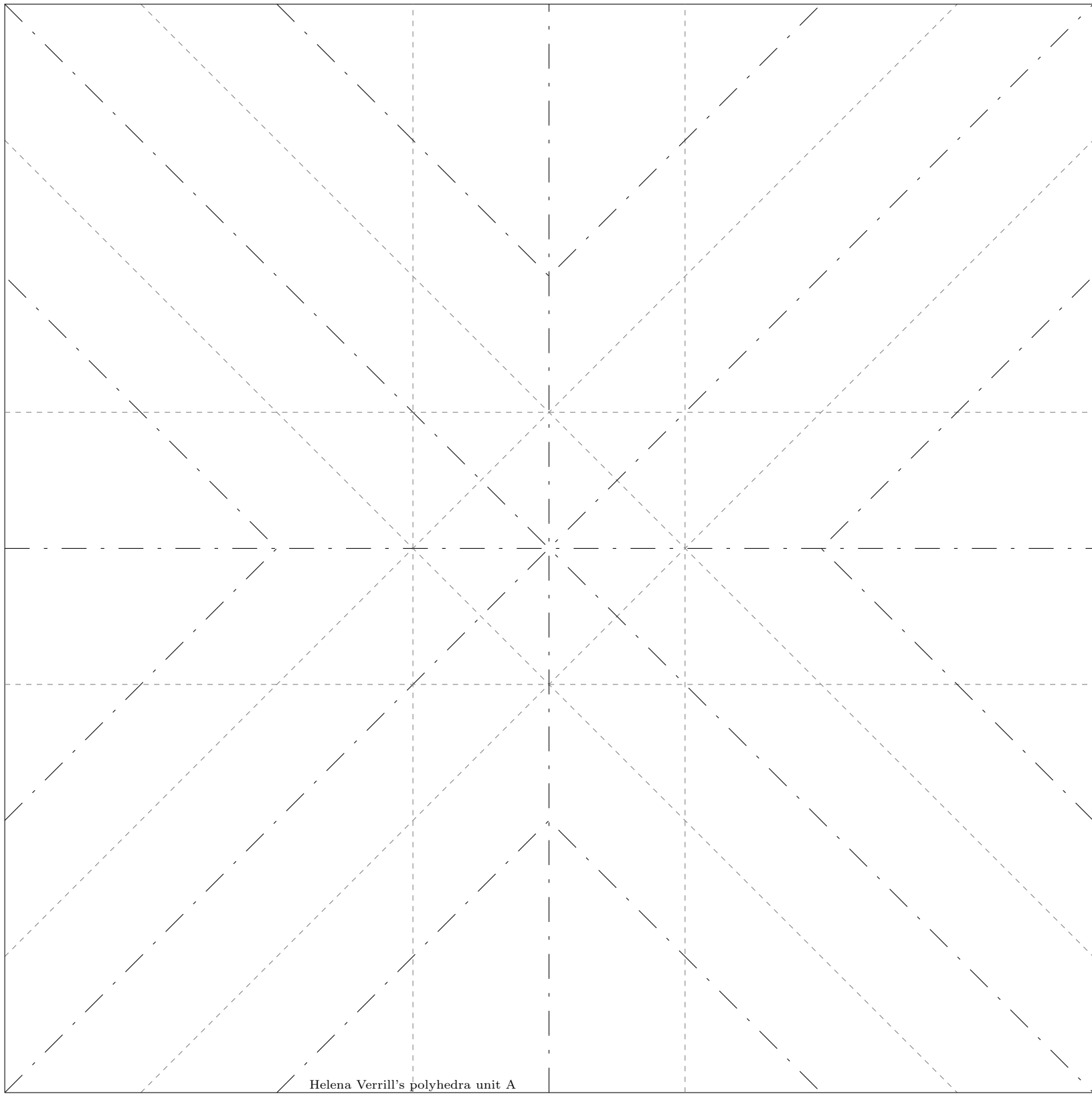
If all vertices have degree 4, as for this unit, and all faces either 3 or 4 sides, then you can use this formula to show that there have to be:

8 triangular faces, the rest are squares

If you want n edges, you need $n/2$ units.
If you want m faces, you need $m - 2$ units.
If you want k vertices, you need k units.

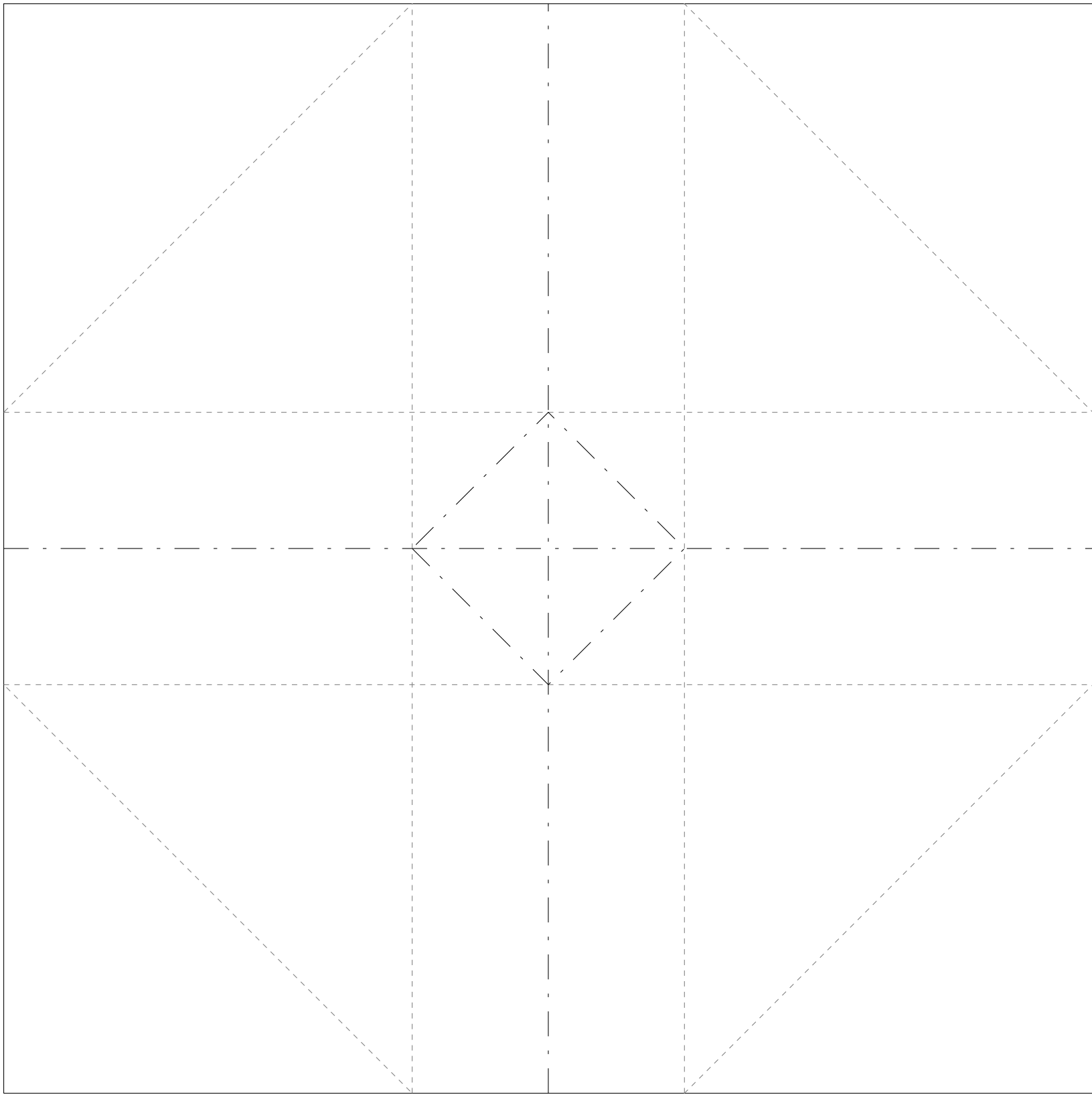
Question:
How many different polyhedra can you make with X units?
Can you make a polyhedron with less than 6 units? with 7 units?

Crease patterns to print on A4 or letter paper:
A: basic polyhedra unit

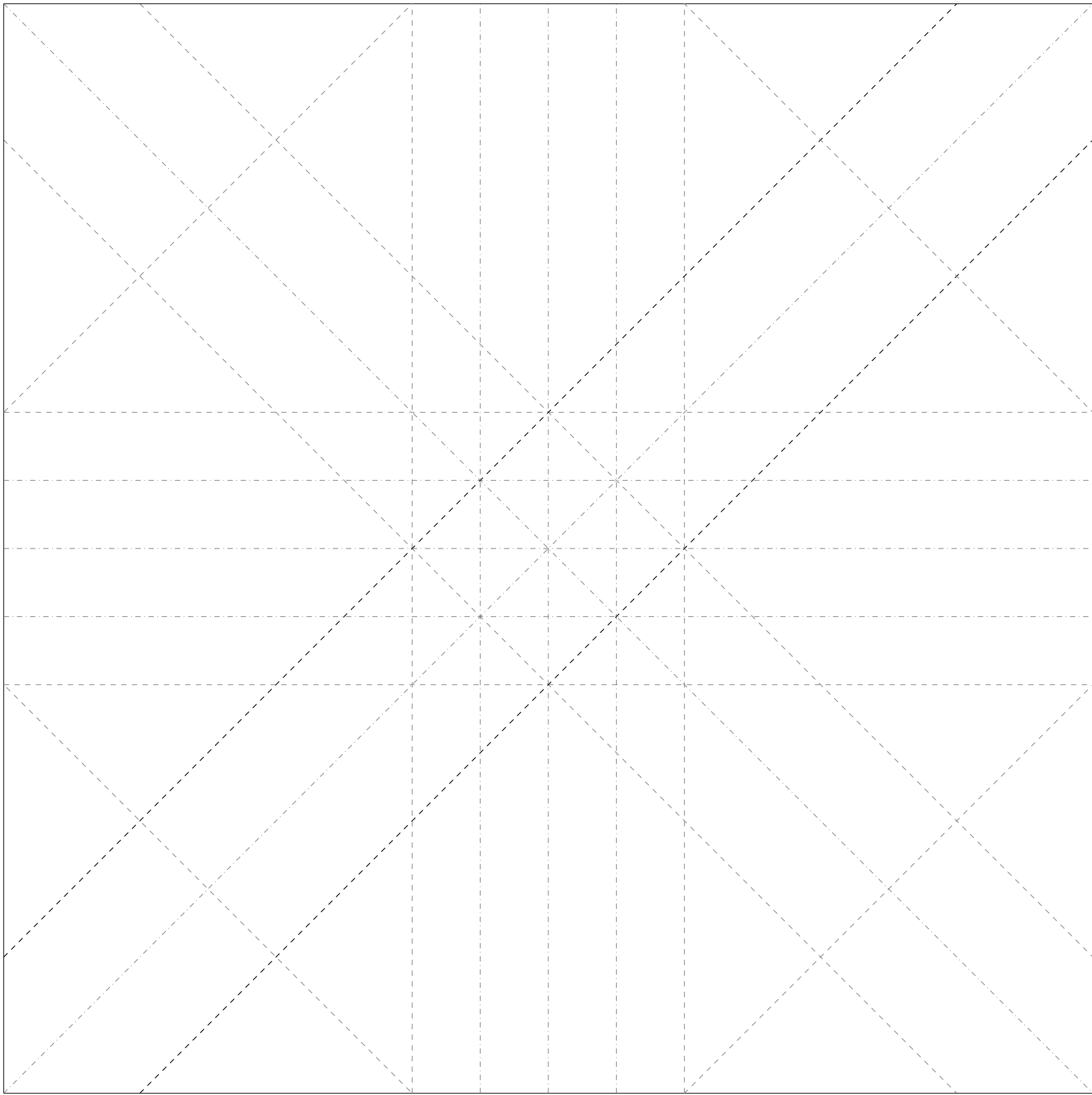


Helena Verrill's polyhedra unit A

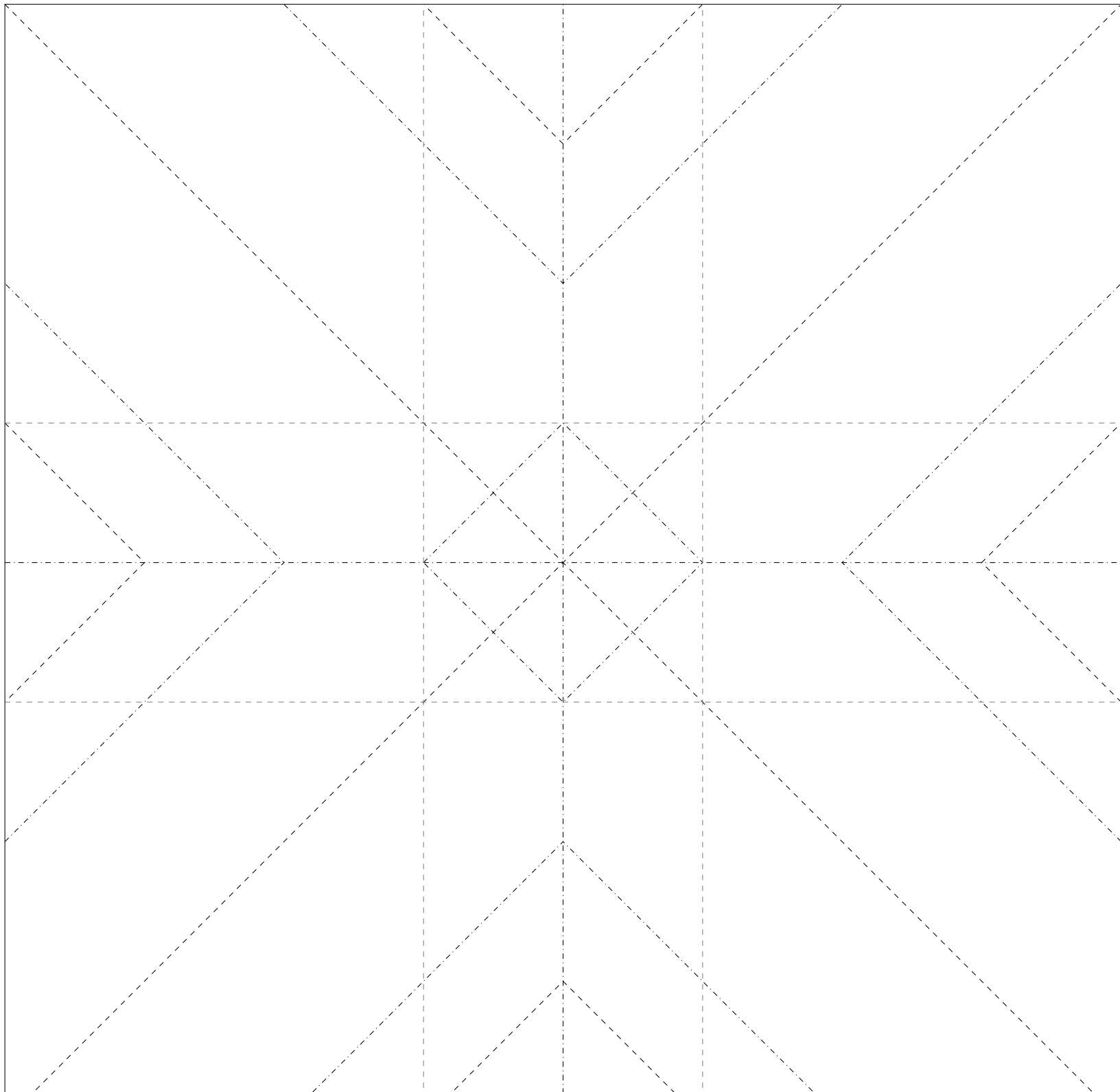
B: variation on polyhedra unit: very simplified version



Alternate version, filled in faces



Alternate version, wide edges, easier to slot together; use paper clips while putting in place:



Unit for tetrahedron: