

Continuous deformation of the Waterbomb base Tesselation

H A Verrill

Bridges 2021, August 2021

WEB: www.mathamaze.co.uk/

IG: @havtofold

Thank you for having me, and thanks to the referees!

What is an origami tessellation?

Some examples:



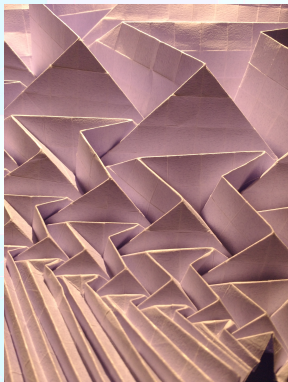
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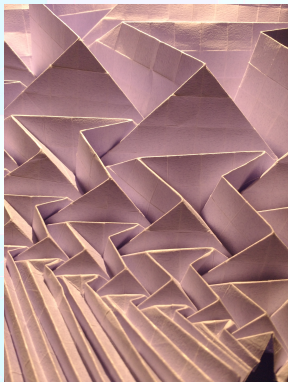
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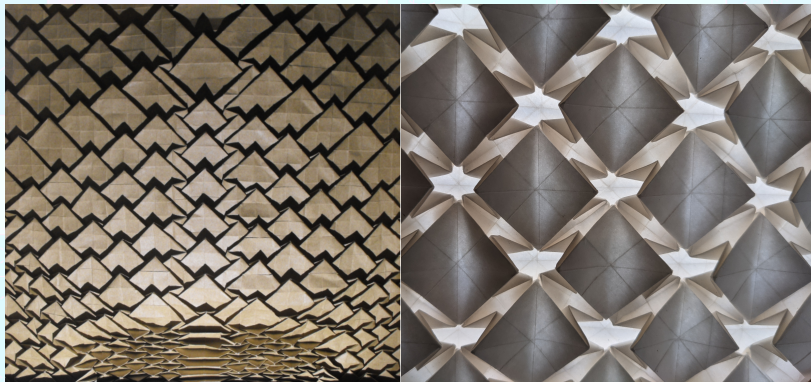
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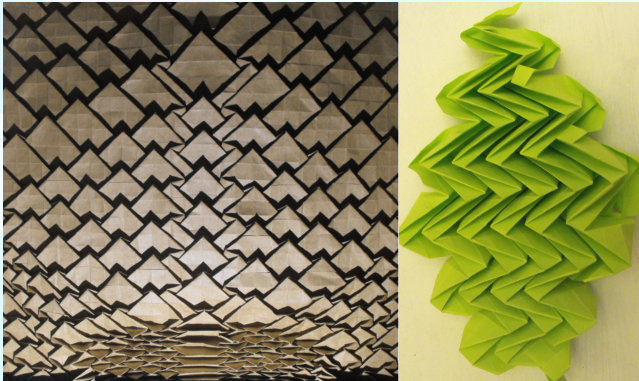
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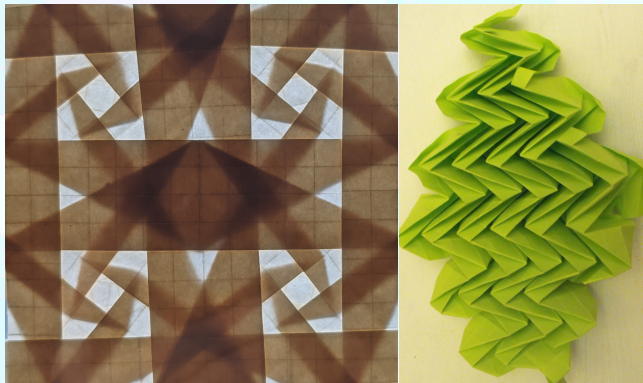
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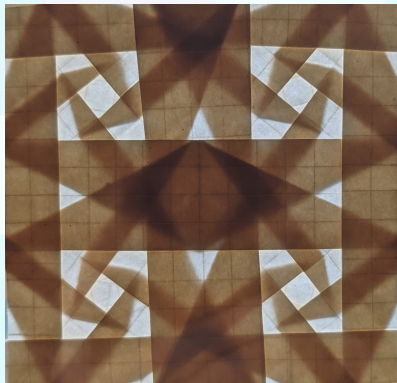
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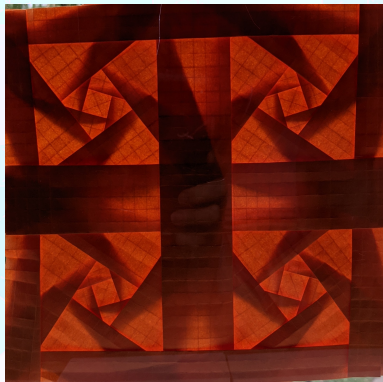
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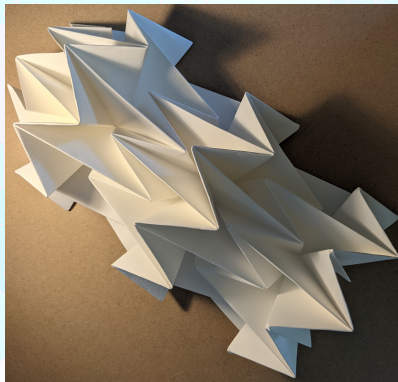
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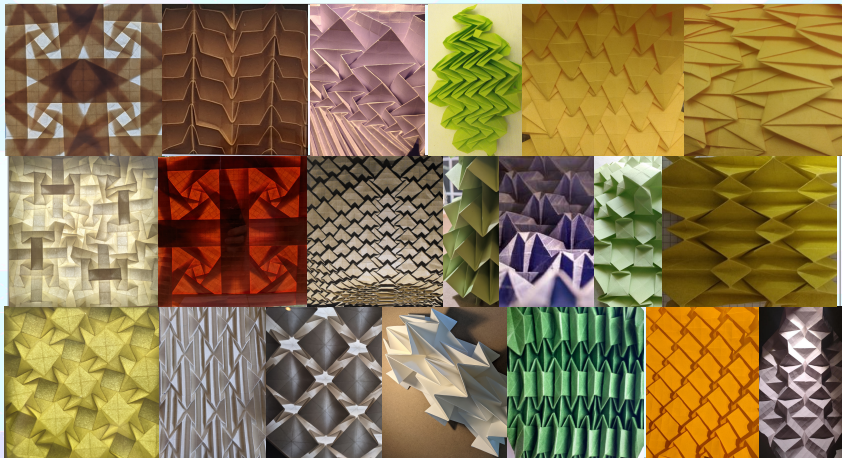
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What is an origami tessellation?

Many things are called origami tessellations



Origami tessellations

- For this talk, an origami tessellation



Origami tessellations

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- is given by a **periodically repeating** crease pattern,

Origami tessellations

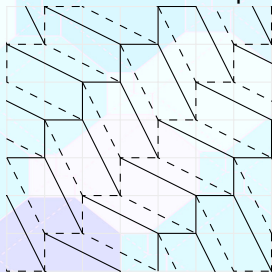
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- which is folded into a **flat** origami.

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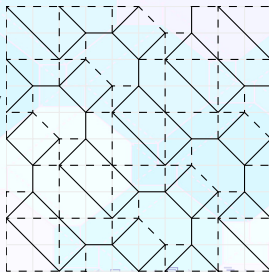
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- Here are two examples:

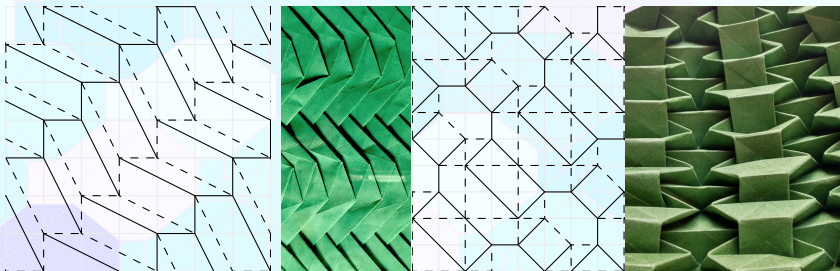


periodically
repeating



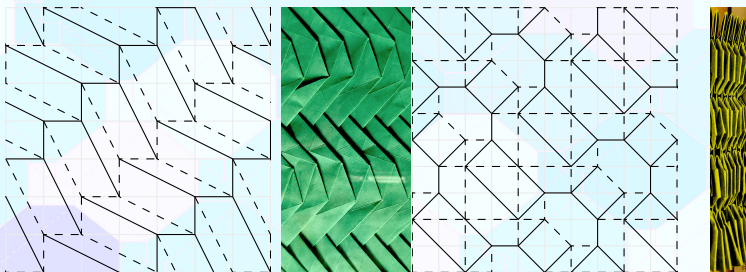
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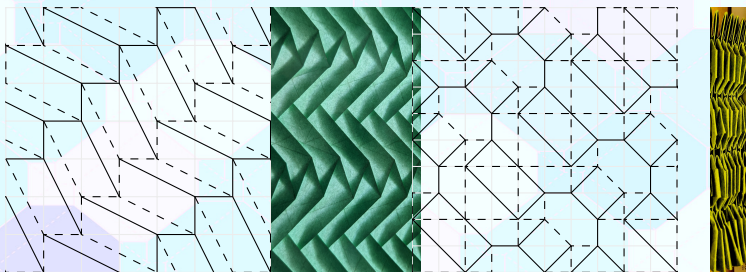
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flat folding origami

- In order to fold flat, an origami must satisfy:

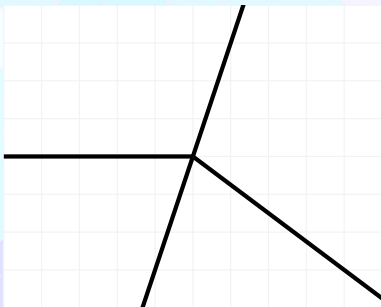


flat folding origami

- In order to fold flat, an origami must satisfy:
- 180° condition at vertices (Kawasaki / Justin / Husimi)

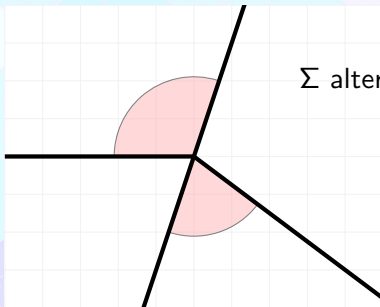
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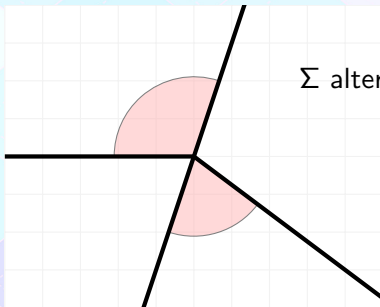
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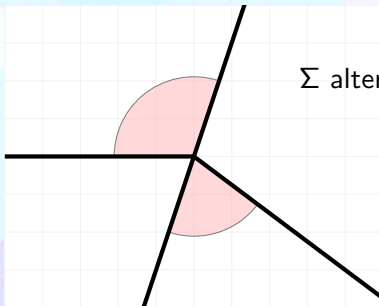


$$\Sigma \text{ alternate angles} = 180^\circ$$

$$\text{e.g., } \text{atan}\left(\frac{1}{3}\right) + \text{atan}\left(\frac{4}{3}\right) = \text{atan}(3)$$

flat folding origami

- In order to fold flat, an origami must satisfy:
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- $|\#(\text{mountain folds}) - \#(\text{valley folds})| = 2$
at every vertex (Maekawa / Justin / Murata)

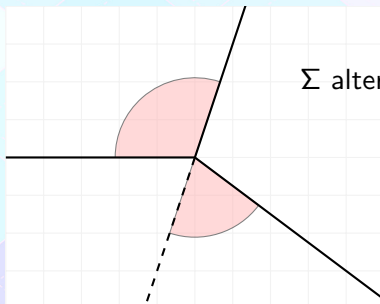


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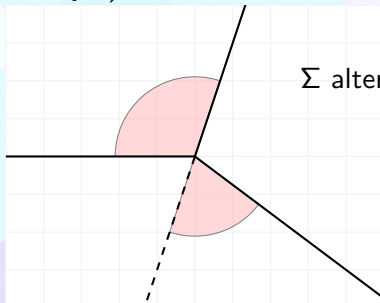


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at every vertex (Maekawa / Justin / Murata)
- complicated conditions about overlapping (NP-complete; Bern & Hayes)

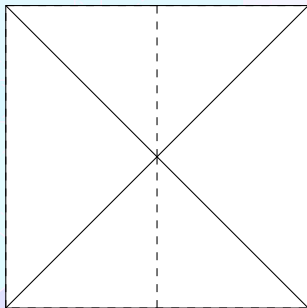


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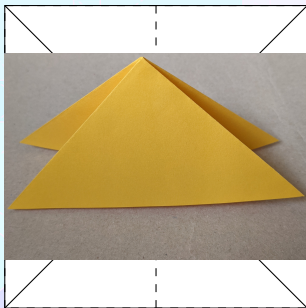
Repetition: Waterbomb base tessellation

Waterbomb base
(traditional)



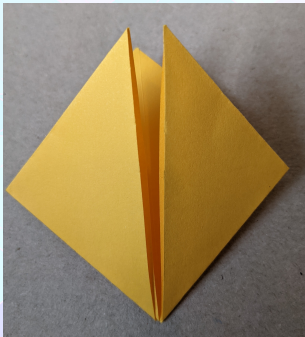
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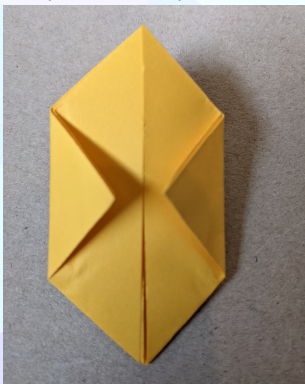
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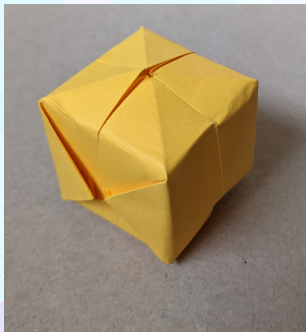
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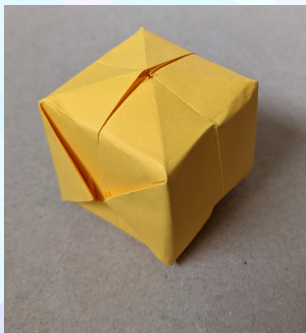
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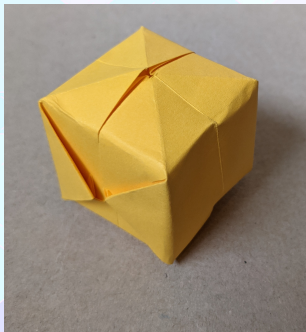


Waterbomb base tessellation
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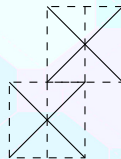


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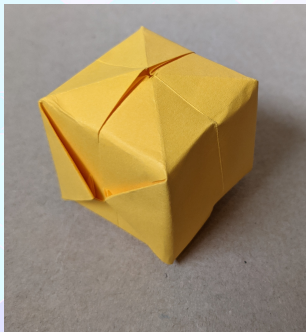


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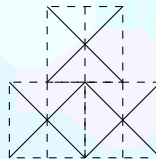


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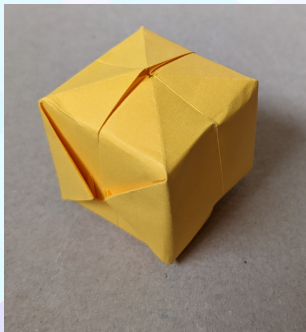


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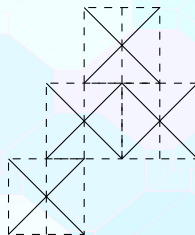


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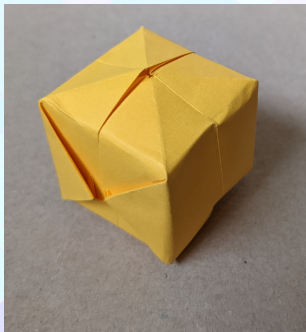


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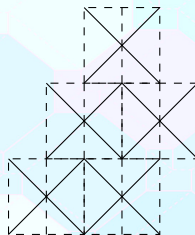


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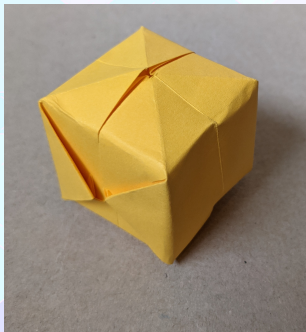


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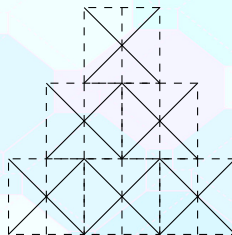


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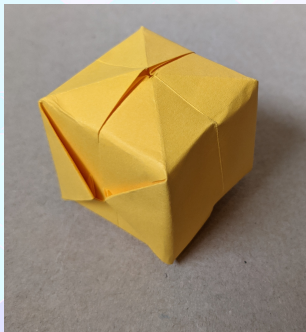


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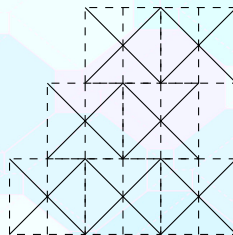


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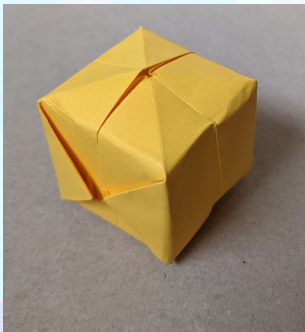


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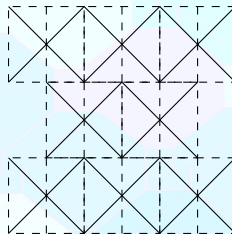


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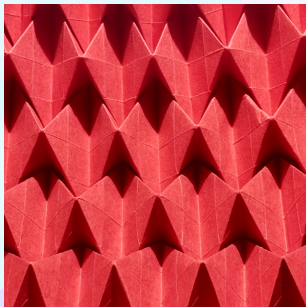


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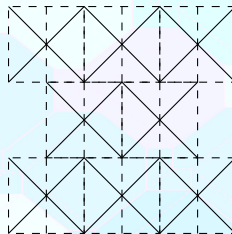


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Folded “front”:

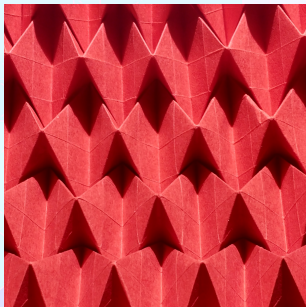


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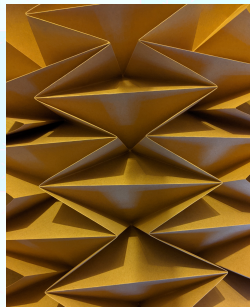
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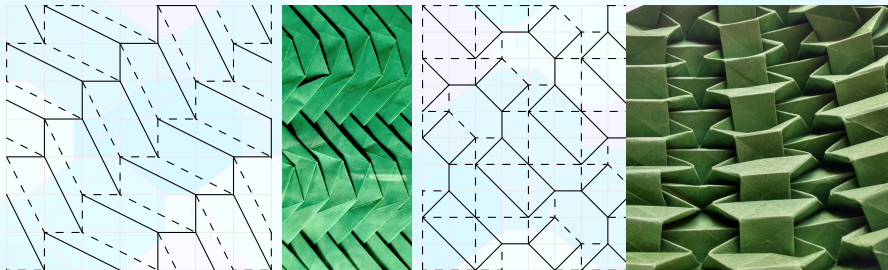
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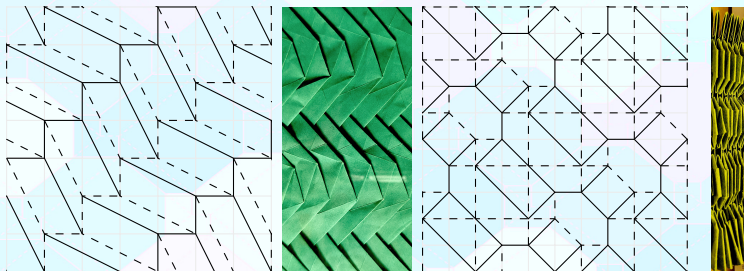
Corrugation or not?

Two tessellations



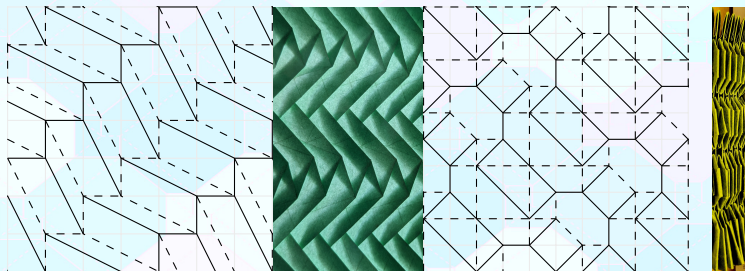
Corrugation or not?

One is not flattened to view - springy; corrugation



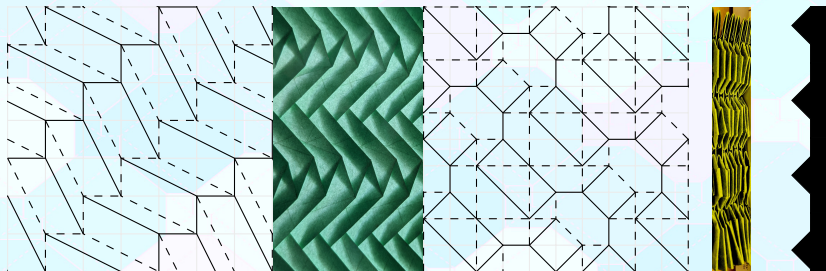
Corrugation or not?

The non corrugation has a better light pattern



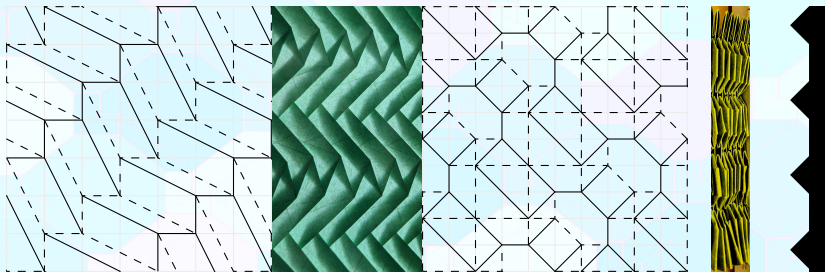
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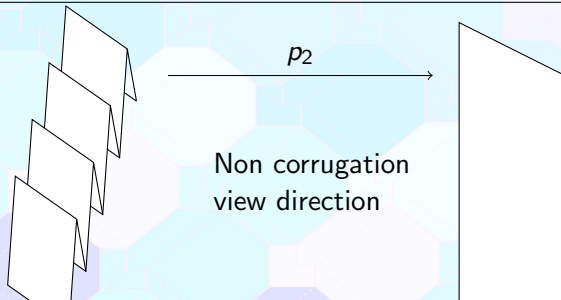
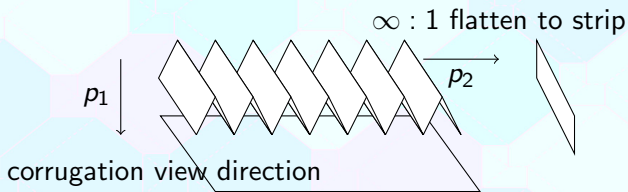


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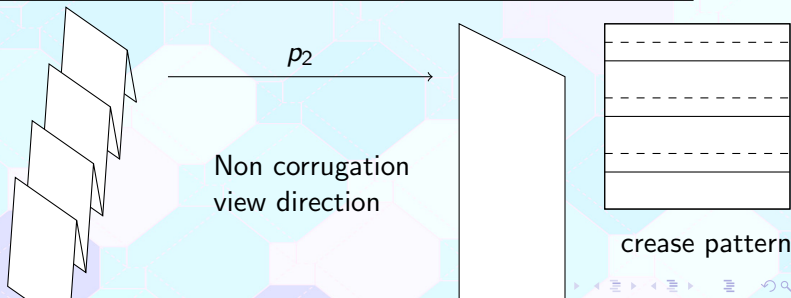
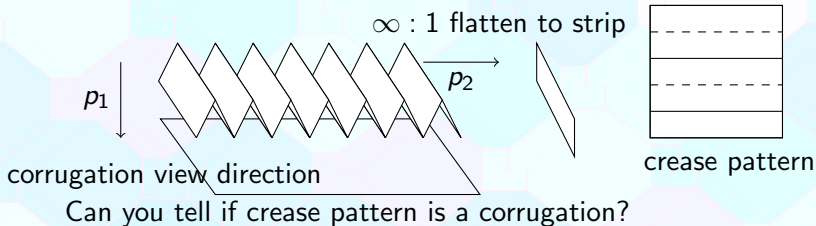
I will show how to continuously pass from certain (limited class of) corrugations to non corrugations



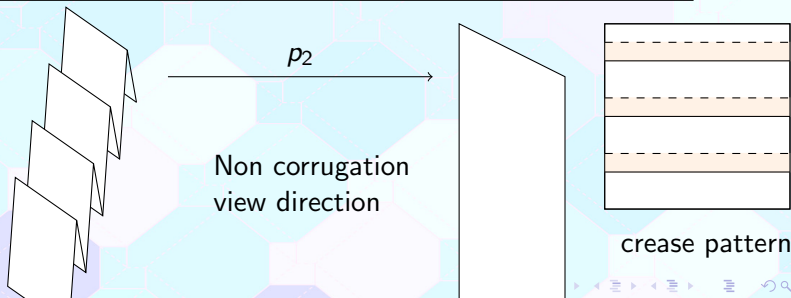
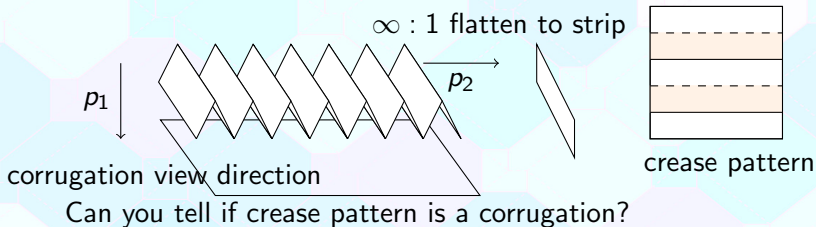
Corrugations - my definition for this talk



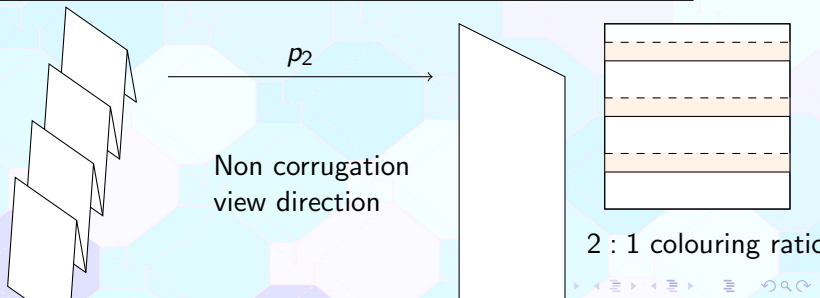
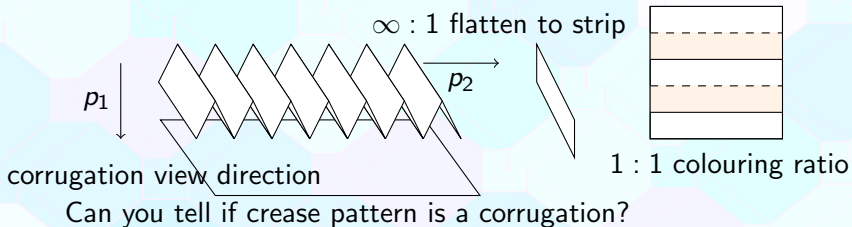
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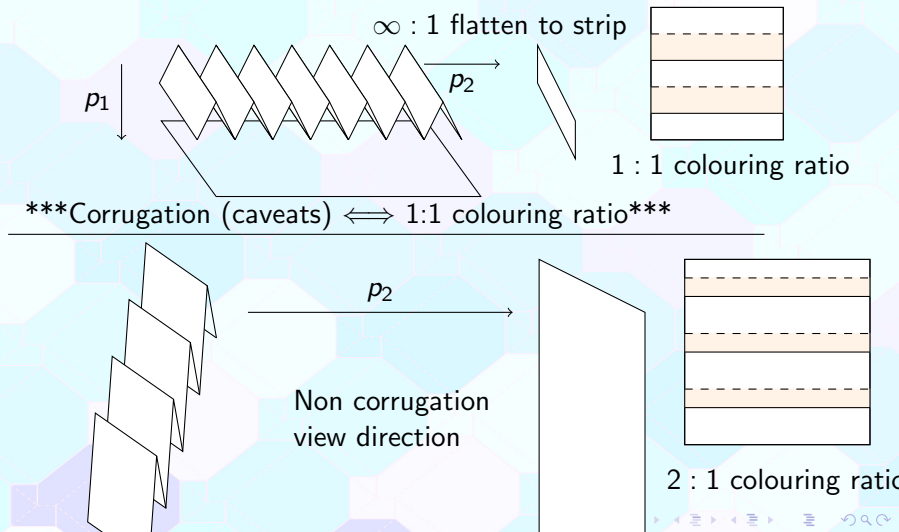
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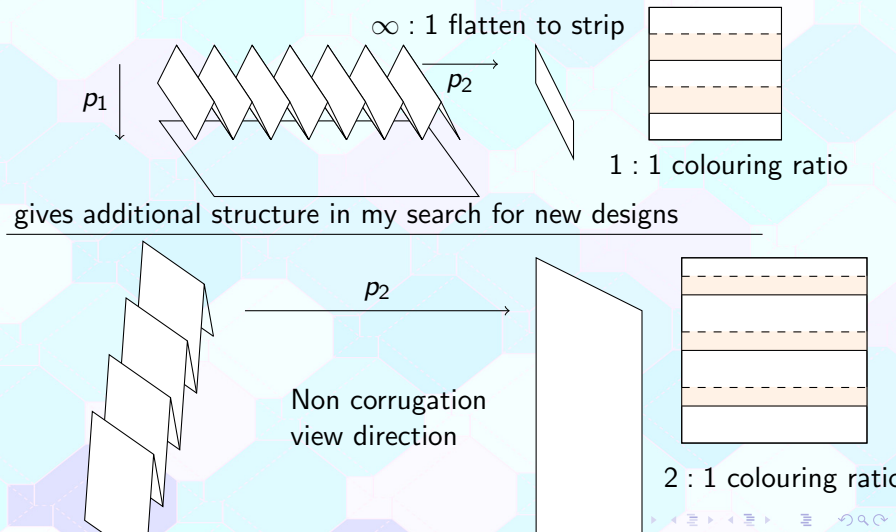
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Examples of deformations of waterbomb base tessellation

Corrugations flattened



Examples of deformations of waterbomb base tessellation

Corrugations unflattened



Examples of deformations of waterbomb base tessellation

non corrugations



The families

On to the continuously varying families of tessellations...



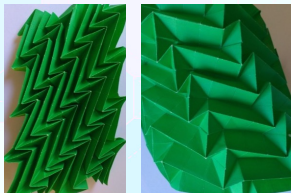
LamiOri variation of the waterbomb base tessellation

Alessandra Lamio's variation:



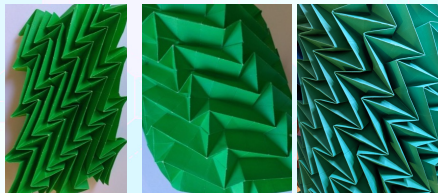
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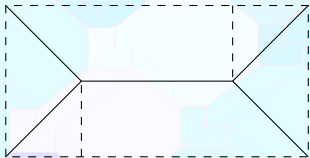
LamiOri variation of the waterbomb base tessellation

Alessandra Lamio's variation: mountain peak becomes a ridge



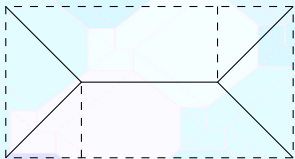
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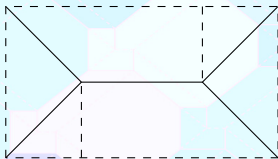
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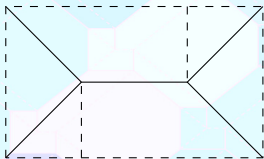
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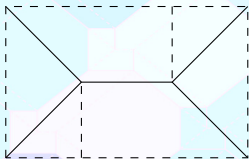
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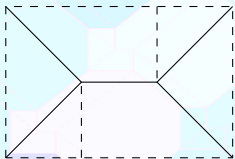
LamiOri variation of the waterbomb base tessellation

Alessandra Lamio's variation: mountain peak becomes a ridge



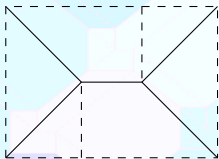
LamiOri variation of the waterbomb base tessellation

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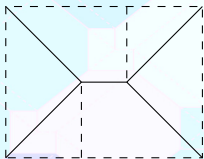
LamiOri variation of the waterbomb base tessellation

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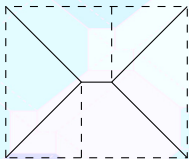
LamiOri variation of the waterbomb base tessellation

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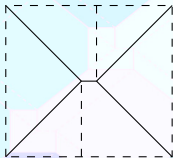
LamiOri variation of the waterbomb base tessellation

Alessandra Lamio's variation: mountain peak becomes a ridge



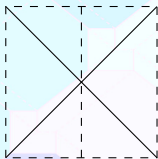
LamiOri variation of the waterbomb base tessellation

Alessandra Lamio's variation: mountain peak becomes a ridge



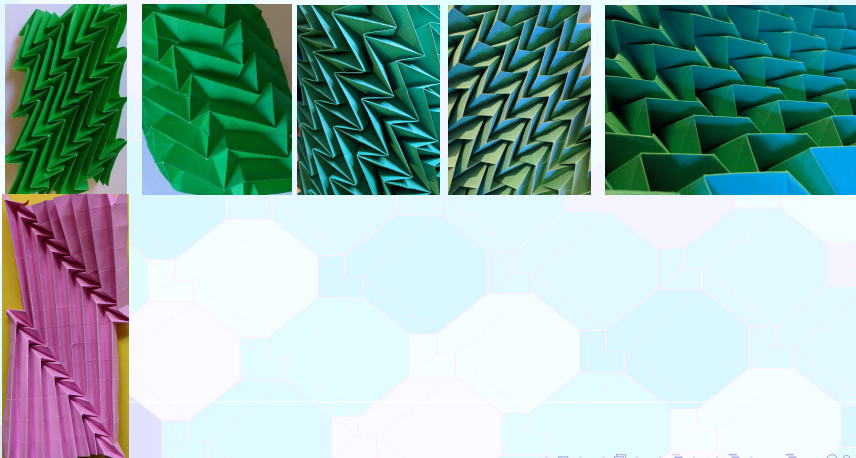
LamiOri variation of the waterbomb base tessellation

Alessandra Lamio's variation: mountain peak becomes a ridge



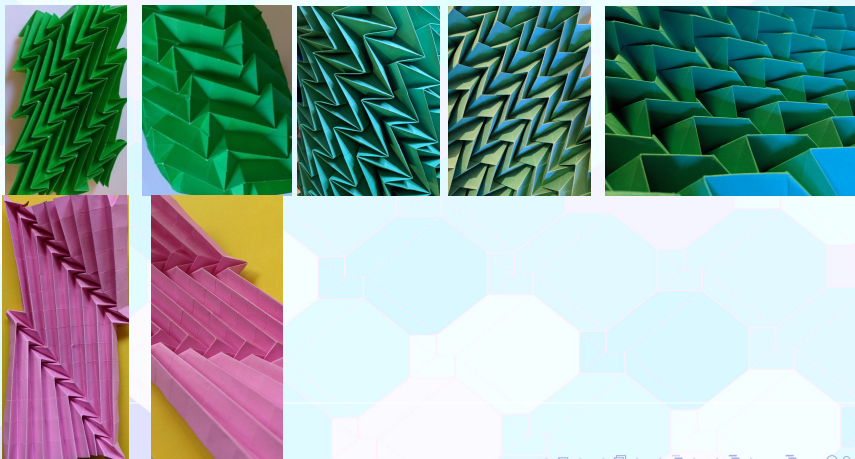
LamiOri variation of the waterbomb base tessellation

Alessandra Lamio's variation: mountain peak becomes a ridge



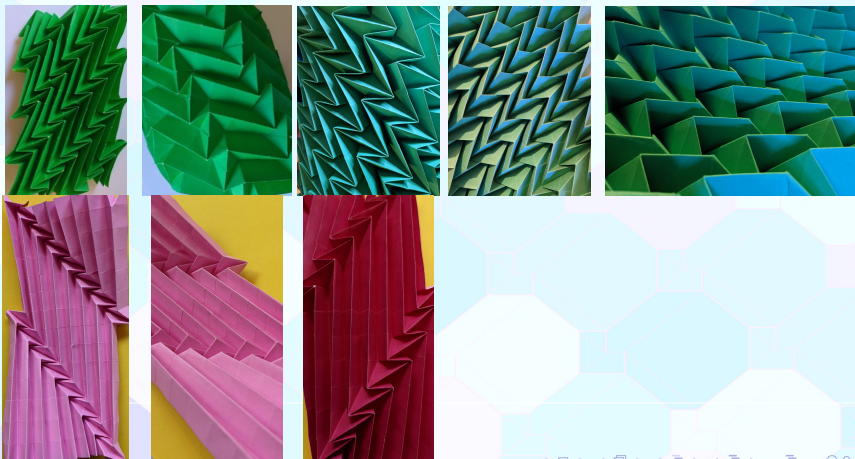
LamiOri variation of the waterbomb base tessellation

Alessandra Lamio's variation: mountain peak becomes a ridge



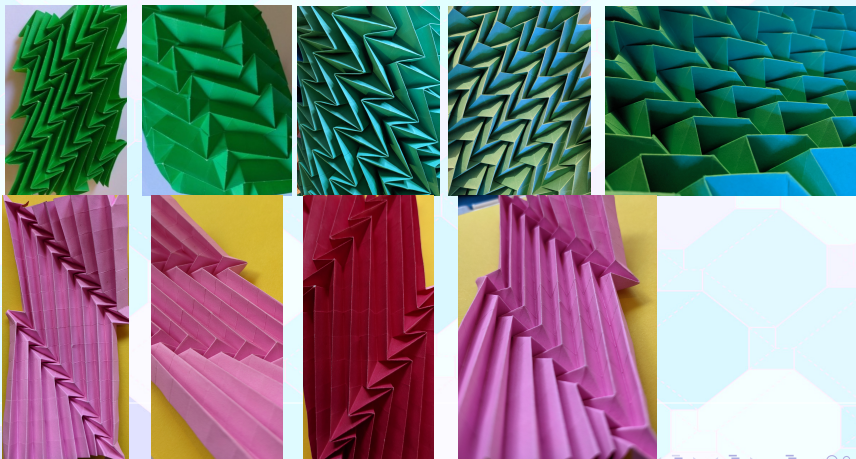
LamiOri variation of the waterbomb base tessellation

Alessandra Lamio's variation: mountain peak becomes a ridge



LamiOri variation of the waterbomb base tessellation

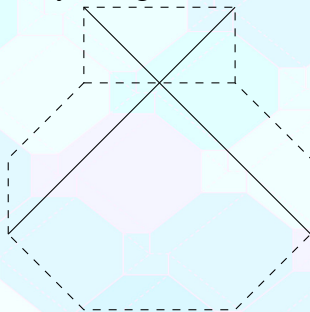
Alessandra Lamio's variation: mountain peak becomes a ridge



Dragon scales tessellation

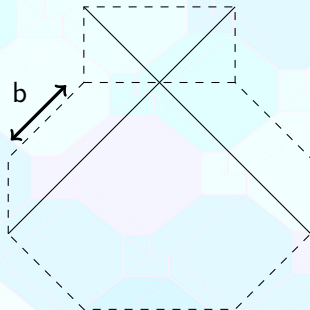


single unit crease pattern; will
show how they fit together later



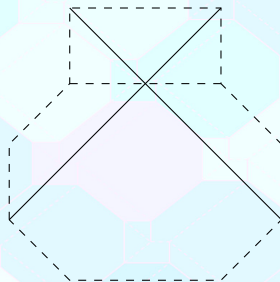
Dragon scales tessellation

this has a parameter

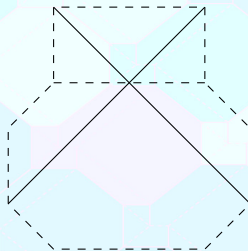


Dragon scales tessellation

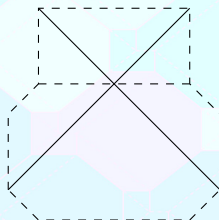
put parameter on an axis



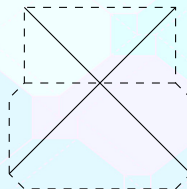
Dragon scales tessellation



Dragon scales tessellation

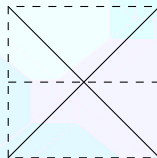


Dragon scales tessellation



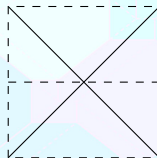
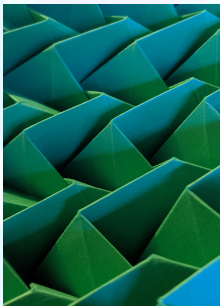
Dragon scales tessellation

We got back to the waterbomb base tessellation!

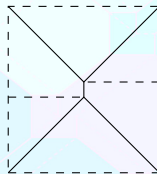
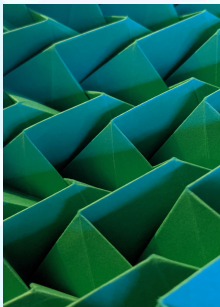


Dragon scales tessellation

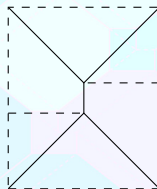
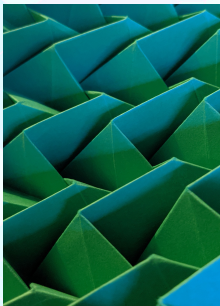
Add another axis!



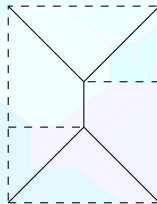
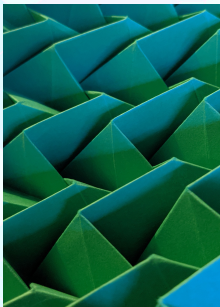
Dragon scales tessellation



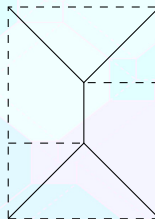
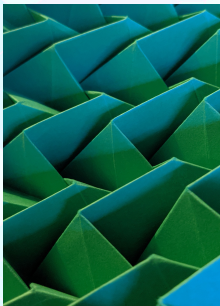
Dragon scales tessellation



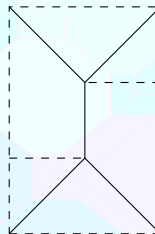
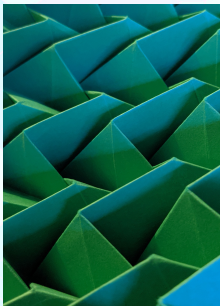
Dragon scales tessellation



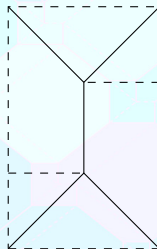
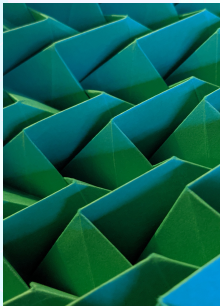
Dragon scales tessellation



Dragon scales tessellation



Dragon scales tessellation



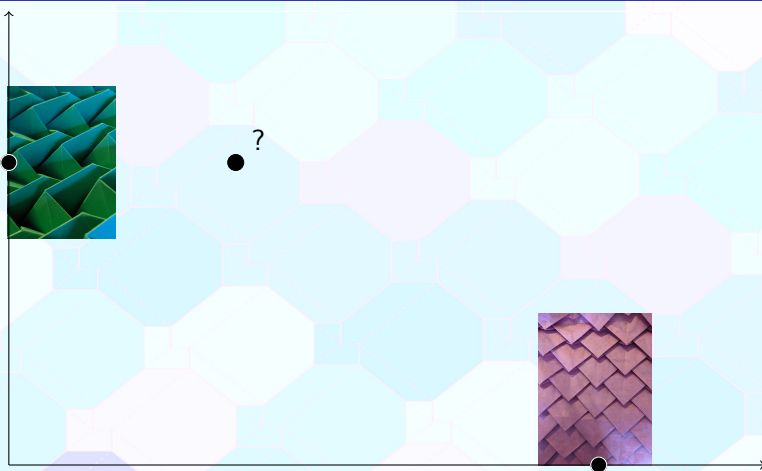
Dragon scales tessellation



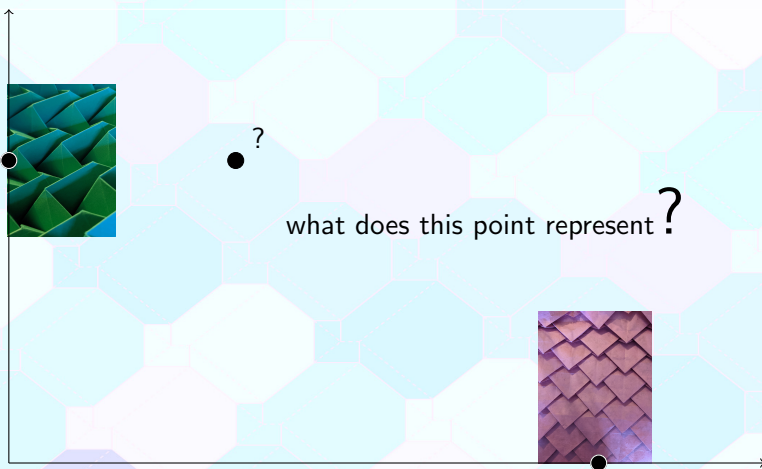
Dragon scales tessellation



Dragon scales tessellation



Dragon scales tessellation



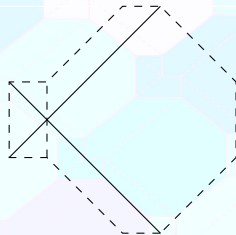
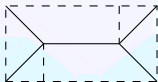
Look at javascript program...

This idea is better illustrated with a javascript program...

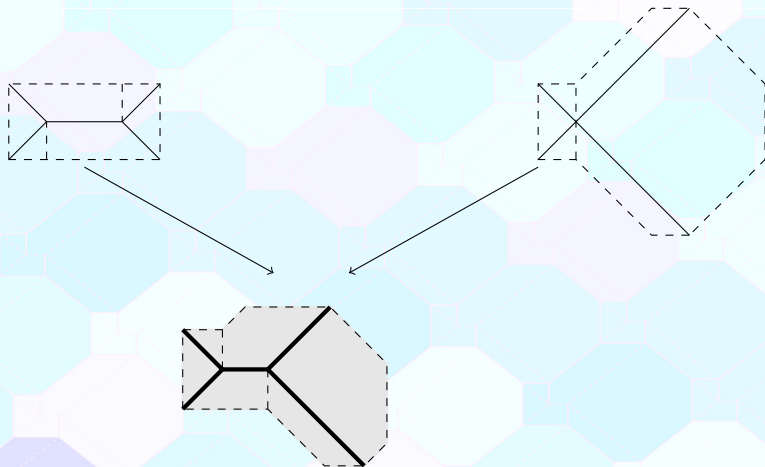
Look at javascript program...

This idea is better illustrated with a javascript program...
<http://www.mathamaze.co.uk/origami/waterbomb/>

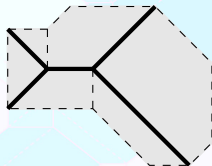
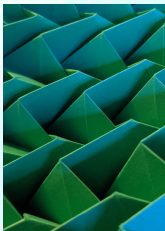
Answer to first question:



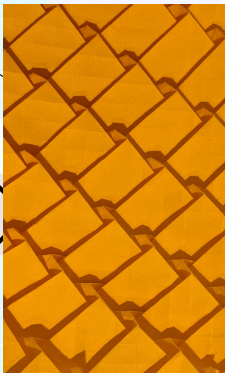
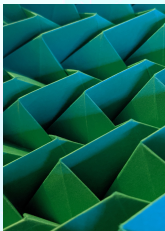
Answer to first question:



Answer to first question:



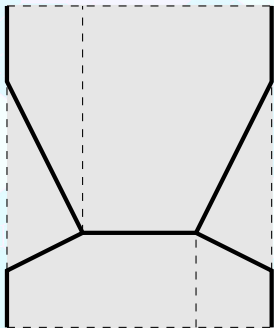
Answer to first question:



Another family

single unit

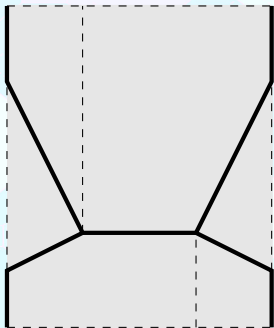
see <http://www.mathamaze.co.uk/origami/origamianimations/waterbombbase1.html> for tessellation



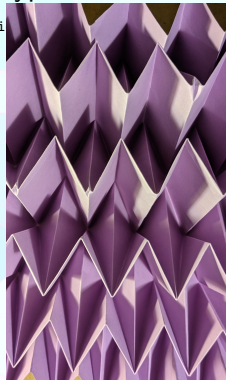
Another family

single unit

see <http://www.mathamaze.co.uk/origami/origamianimation>



typical member of family

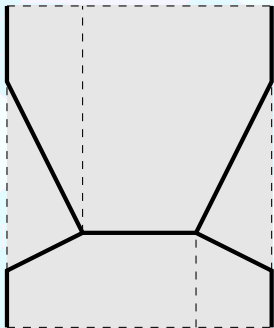


tessellation

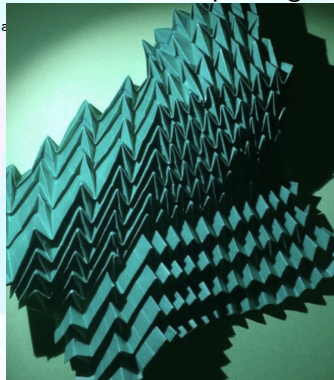
Another family

single unit

see <http://www.mathamaze.co.uk/origami/origami>



different members put together

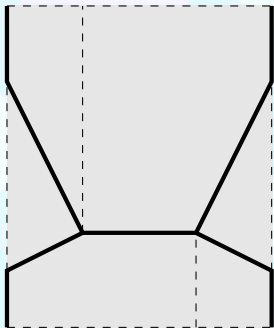


tion

Another family

single unit

see <http://www.mathamaze.co.uk/origami/origamianim>



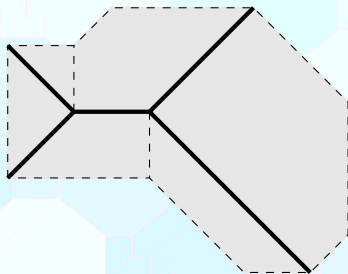
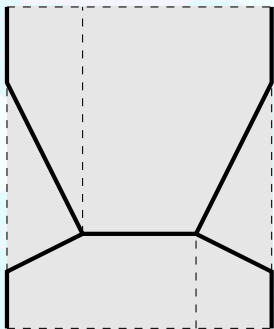
with extra horizontal crease

tessellation



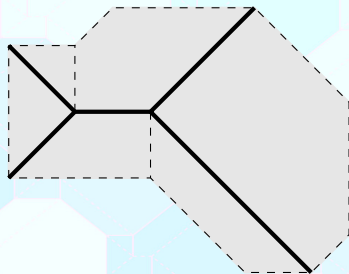
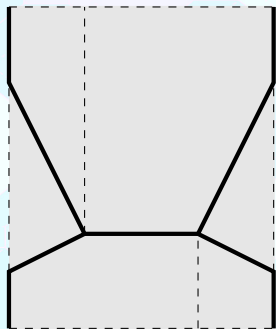
Another family

Might not be possible to combine these two families



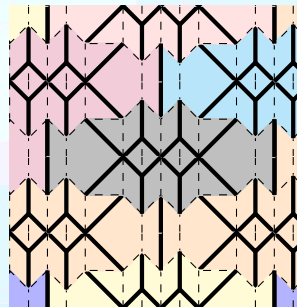
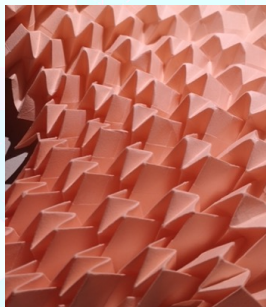
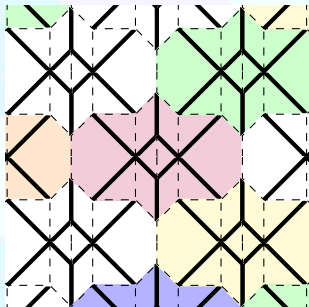
Another family

Might not be possible to combine these two families

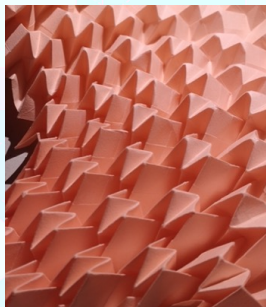
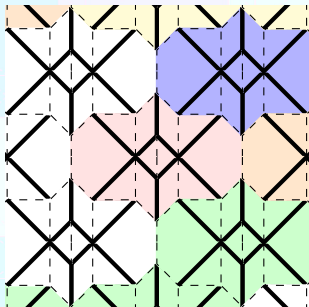


When can families be combined?

Multipeak deformation

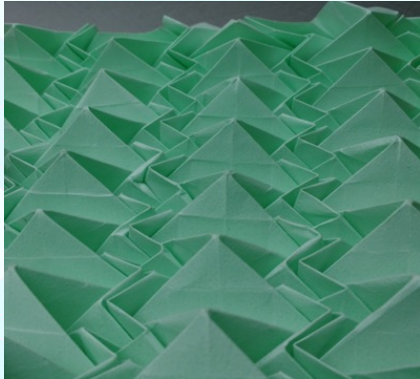
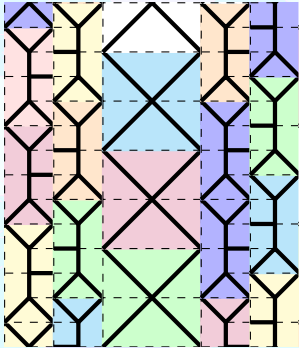


Multipeak deformation



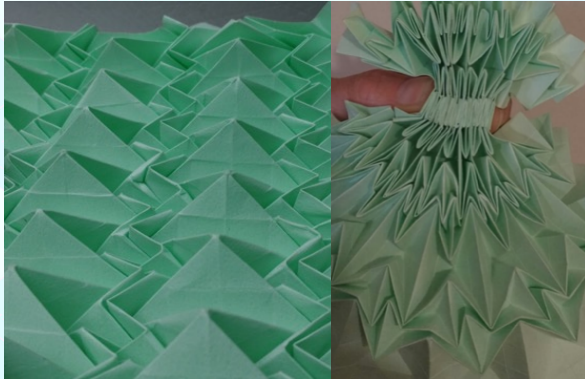
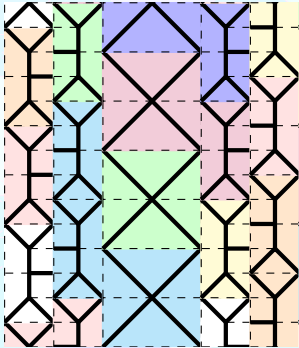
Technique applied to a corrugation which is not a deformation of the waterbomb base tessellation

One waterbomb base unit is twice the size of the other



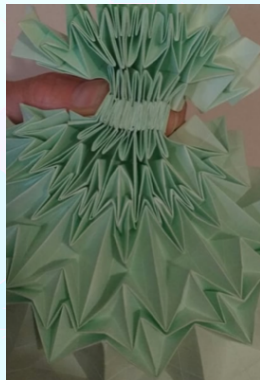
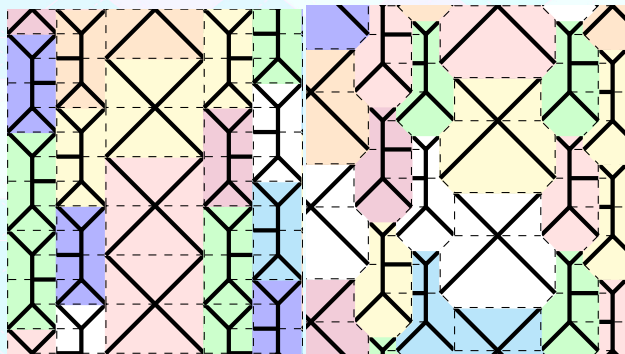
Technique applied to a corrugation which is not a deformation of the waterbomb base tessellation

One waterbomb base unit is twice the size of the other



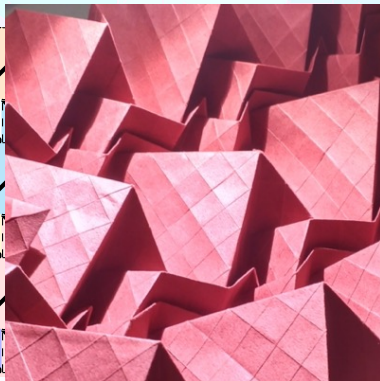
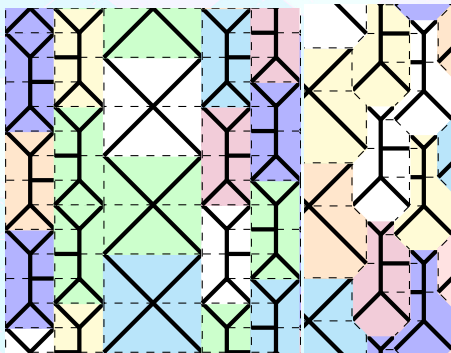
Technique applied to a corrugation which is not a deformation of the waterbomb base tessellation

Apply the dragon scale deformation



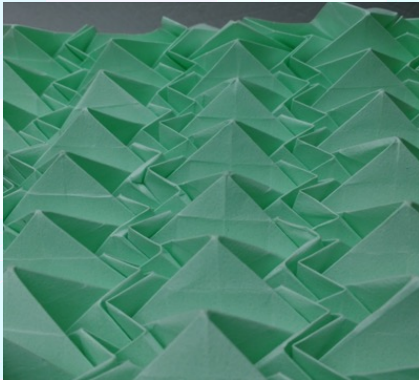
Technique applied to a corrugation which is not a deformation of the waterbomb base tessellation

Apply the dragon scale deformation



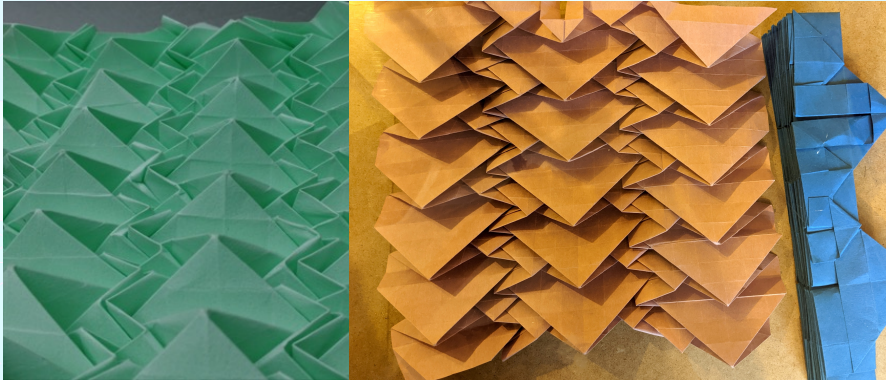
Technique applied to a corrugation which is not a deformation of the waterbomb base tessellation

They look very similar



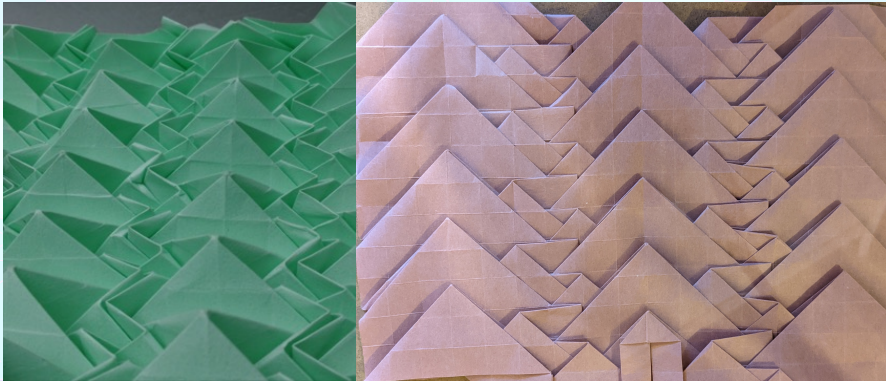
Technique applied to a corrugation which is not a deformation of the waterbomb base tessellation

But one is much bulkier than the other when flat



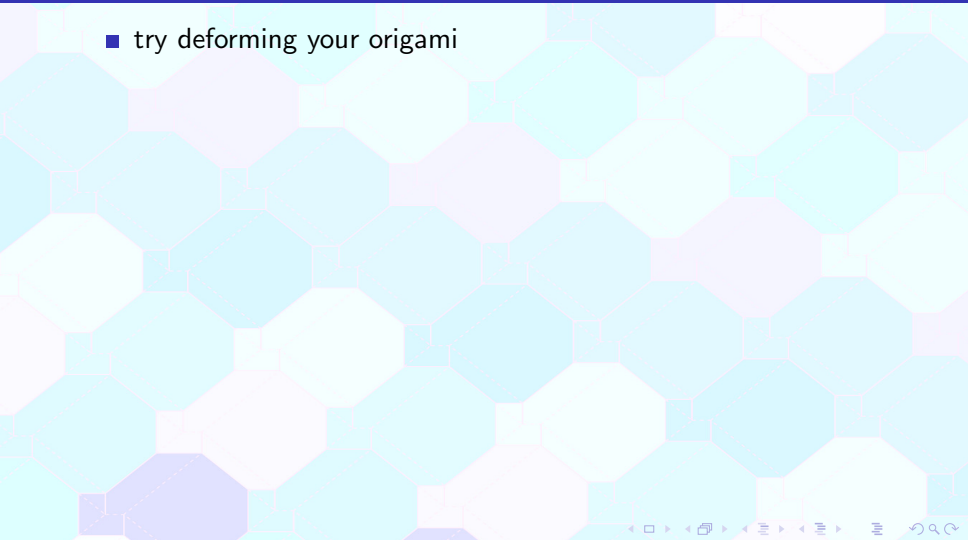
Technique applied to a corrugation which is not a deformation of the waterbomb base tessellation

In the deformation, the pattern can be seen when flat



Recap

- try deforming your origami



Recap

- try deforming your origami
- try combining two kinds of deformations

Recap

- try deforming your origami
- try combining two kinds of deformations
- try spreading out your corrugation to form a flatter tessellation

Recap

- try deforming your origami
- try combining two kinds of deformations
- try spreading out your corrugation to form a flatter tessellation
- combine different units from same family

